R the resistance, then R$x$ will be the resisting force. In steady currents the electromotive force just balances the resisting force, but in variable currents the resultant force $\xi = \mathrm{R}x$ is expended in increasing the "electromagnetic momentum," using the word momentum merely to express that which is generated by a force acting during a time, that is, a velocity existing in a body.

In the case of electric currents, the force in action is not ordinary mechanical force, at least we are not as yet able to measure it as common force, but we call it electromotive force, and the body moved is not merely the electricity in the conductor, but something outside the conductor, and capable of being affected by other conductors in the neighbourhood carrying currents. In this it resembles rather the reduced momentum of a driving-point of a machine as influenced by its mechanical connexions, than that of a simple moving body like a cannon ball, or water in a tube.

### *Electromagnetic Relations of two Conducting Circuits.*

(28.) In the case of two conducting circuits, A and B, we shall assume that the electromagnetic momentum belonging to A is

$$\mathrm{L}x + \mathrm{M}y,$$

and that belonging to B,

$$\mathrm{M}x + \mathrm{N}y,$$

where L, M, N correspond to the same quantities in the dynamical illustration, except that they are supposed to be capable of variation when the conductors A or B are moved.

Then the equation of the current $x$ in A will be

$$\xi = \mathrm{R}x + \frac{d}{dt}(\mathrm{L}x + \mathrm{M}y), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and that of $y$ in B

$$\eta = \mathrm{S}y + \frac{d}{dt}(\mathrm{M}x + \mathrm{N}y), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $\xi$ and $\eta$ are the electromotive forces, $x$ and $y$ the currents, and R and S the resistances in A and B respectively.

### *Induction of one Current by another.*

(29) Case 1st. Let there be no electromotive force on B, except that which arises from the action of A, and let the current of A increase from 0 to the value $x$, then

$$\mathrm{S}y + \frac{d}{dt}(\mathrm{M}x + \mathrm{N}y) = 0,$$

whence

$$\mathrm{Y} = \int_0^t y\,dt = -\frac{\mathrm{M}}{\mathrm{S}}x,$$

that is, a quantity of electricity Y, being the total induced current, will flow through B when $x$ rises from 0 to $x$. This is induction by variation of the current in the primary conductor. When M is positive, the induced current due to increase of the primary current is negative.

*Induction by Motion of Conductor.*

(30) Case 2nd. Let $x$ remain constant, and let M change from M to M′, then

$$Y=-\frac{M'-M}{S}x;$$

so that if M is increased, which it will be by the primary and secondary circuits approaching each other, there will be a negative induced current, the total quantity of electricity passed through B being Y.

This is induction by the relative motion of the primary and secondary conductors.

*Equation of Work and Energy.*

(31) To form the equation between work done and energy produced, multiply (1) by $x$ and (2) by $y$, and add

$$\xi x+\eta y=Rx^2+Sy^2+x\frac{d}{dt}(Lx+My)+y\frac{d}{dt}(Mx+Ny). \quad . \quad . \quad . \quad . \quad (8)$$

Here $\xi x$ is the work done in unit of time by the electromotive force $\xi$ acting on the current $x$ and maintaining it, and $\eta y$ is the work done by the electromotive force $\eta$. Hence the left-hand side of the equation represents the work done by the electromotive forces in unit of time.

*Heat produced by the Current.*

(32) On the other side of the equation we have, first,

$$Rx^2+Sy^2=H, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which represents the work done in overcoming the resistance of the circuits in unit of time. This is converted into heat. The remaining terms represent work not converted into heat. They may be written

$$\tfrac{1}{2}\frac{d}{dt}(Lx^2+2Mxy+Ny^2)+\tfrac{1}{2}\frac{dL}{dt}x^2+\frac{dM}{dt}xy+\tfrac{1}{2}\frac{dN}{dt}y^2.$$

*Intrinsic Energy of the Currents.*

(33) If L, M, N are constant, the whole work of the electromotive forces which is not spent against resistance will be devoted to the development of the currents. The whole intrinsic energy of the currents is therefore

$$\tfrac{1}{2}Lx^2+Mxy+\tfrac{1}{2}Ny^2=E. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This energy exists in a form imperceptible to our senses, probably as actual motion, the seat of this motion being not merely the conducting circuits, but the space surrounding them.

*Mechanical Action between Conductors.*

(34) The remaining terms,

$$\frac{1}{2}\frac{dL}{dt}x^2+\frac{dM}{dt}xy+\frac{1}{2}\frac{dN}{dt}y^2=W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

represent the work done in unit of time arising from the variations of L, M, and N, or, what is the same thing, alterations in the form and position of the conducting circuits A and B.

Now if work is done when a body is moved, it must arise from ordinary mechanical force acting on the body while it is moved. Hence this part of the expression shows that there is a mechanical force urging every part of the conductors themselves in that direction in which L, M, and N will be most increased.

The existence of the electromagnetic force between conductors carrying currents is therefore a direct consequence of the joint and independent action of each current on the electromagnetic field. If A and B are allowed to approach a distance $ds$, so as to increase M from M to M′ while the currents are $x$ and $y$, then the work done will be

$$(M'-M)xy,$$

and the force in the direction of $ds$ will be

$$\frac{dM}{ds}xy, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and this will be an attraction if $x$ and $y$ are of the same sign, and if M is increased as A and B approach.

It appears, therefore, that if we admit that the unresisted part of electromotive force goes on as long as it acts, generating a self-persistent state of the current, which we may call (from mechanical analogy) its electromagnetic momentum, and that this momentum depends on circumstances external to the conductor, then both induction of currents and electromagnetic attractions may be proved by mechanical reasoning.

What I have called electromagnetic momentum is the same quantity which is called by Faraday* the electrotonic state of the circuit, every change of which involves the action of an electromotive force, just as change of momentum involves the action of mechanical force.

If, therefore, the phenomena described by Faraday in the Ninth Series of his Experimental Researches were the only known facts about electric currents, the laws of Ampère relating to the attraction of conductors carrying currents, as well as those of Faraday about the mutual induction of currents, might be deduced by mechanical reasoning.

In order to bring these results within the range of experimental verification, I shall next investigate the case of a single current, of two currents, and of the six currents in the electric balance, so as to enable the experimenter to determine the values of L, M, N.

* Experimental Researches, Series I. 60, &c.

*Case of a single Circuit.*

(35) The equation of the current $x$ in a circuit whose resistance is R, and whose coefficient of self-induction is L, acted on by an external electromotive force $\xi$, is

$$\xi - Rx = \frac{d}{dt} Lx. \qquad (13)$$

When $\xi$ is constant, the solution is of the form

$$x = b + (a-b)e^{-\frac{R}{L}t},$$

where $a$ is the value of the current at the commencement, and $b$ is its final value.

The total quantity of electricity which passes in time $t$, where $t$ is great, is

$$\int_0^t x dt = bt + (a-b)\frac{L}{R}. \qquad (14)$$

The value of the integral of $x^2$ with respect to the time is

$$\int_0^t x^2 dt = b^2 t + (a-b)\frac{L}{R}\left(\frac{3b+a}{2}\right). \qquad (15)$$

The actual current changes gradually from the initial value $a$ to the final value $b$, but the values of the integrals of $x$ and $x^2$ are the same as if a steady current of intensity $\frac{1}{2}(a+b)$ were to flow for a time $2\frac{L}{R}$, and were then succeeded by the steady current $b$. The time $2\frac{L}{R}$ is generally so minute a fraction of a second, that the effects on the galvanometer and dynamometer may be calculated as if the impulse were instantaneous.

If the circuit consists of a battery and a coil, then, when the circuit is first completed, the effects are the same as if the current had only half its final strength during the time $2\frac{L}{R}$. This diminution of the current, due to induction, is sometimes called the counter-current.

(36) If an additional resistance $r$ is suddenly thrown into the circuit, as by breaking contact, so as to force the current to pass through a thin wire of resistance $r$, then the original current is $a=\frac{\xi}{R}$, and the final current is $b=\frac{\xi}{R+r}$.

The current of induction is then $\frac{1}{2}\xi\frac{2R+r}{R(R+r)}$, and continues for a time $2\frac{L}{R+r}$. This current is greater than that which the battery can maintain in the two wires R and $r$, and may be sufficient to ignite the thin wire $r$.

When contact is broken by separating the wires in air, this additional resistance is given by the interposed air, and since the electromotive force across the new resistance is very great, a spark will be forced across.

If the electromotive force is of the form $E \sin pt$, as in the case of a coil revolving in a magnetic field, then

$$x=\frac{E}{\varrho}\sin(pt-\alpha),$$

where $\varrho^2=R^2+L^2p^2$, and $\tan\alpha=\frac{Lp}{R}$.

*Case of two Circuits.*

(37) Let R be the primary circuit and S the secondary circuit, then we have a case similar to that of the induction coil.

The equations of currents are those marked A and B, and we may here assume L, M, N as constant because there is no motion of the conductors. The equations then become

$$\left.\begin{aligned} Rx+L\frac{dx}{dt}+M\frac{dy}{dt}&=\xi, \\ Sy+M\frac{dx}{dt}+N\frac{dy}{dt}&=0. \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (13^*)$$

To find the total quantity of electricity which passes, we have only to integrate these equations with respect to $t$; then if $x_0$, $y_0$ be the strengths of the currents at time 0, and $x_1$, $y_1$ at time $t$, and if X, Y be the quantities of electricity passed through each circuit during time $t$,

$$\left.\begin{aligned} X&=\frac{1}{R}\{\xi t+L(x_0-x_1)+M(y_0-y_1)\}, \\ Y&=\frac{1}{S}\{M(x_0-x_1)+N(y_0-y_1)\}. \end{aligned}\right\} \quad \ldots \ldots \quad (14^*)$$

When the circuit R is completed, then the total currents up to time $t$, when $t$ is great, are found by making

$$x_0=0, \quad x_1=\frac{\xi}{R}, \quad y_0=0, \quad y_1=0;$$

then

$$X=x_1\left(t-\frac{L}{R}\right), \quad Y=-\frac{M}{S}x_1. \quad \ldots \ldots \ldots \quad (15^*)$$

The value of the total counter-current in R is therefore independent of the secondary circuit, and the induction current in the secondary circuit depends only on M, the coefficient of induction between the coils, S the resistance of the secondary coil, and $x_1$ the final strength of the current in R.

When the electromotive force $\xi$ ceases to act, there is an extra current in the primary circuit, and a positive induced current in the secondary circuit, whose values are equal and opposite to those produced on making contact.

(38) All questions relating to the total quantity of transient currents, as measured by the impulse given to the magnet of the galvanometer, may be solved in this way without the necessity of a complete solution of the equations. The heating effect of

the current, and the impulse it gives to the suspended coil of WEBER's dynamometer, depend on the square of the current at every instant during the short time it lasts. Hence we must obtain the solution of the equations, and from the solution we may find the effects both on the galvanometer and dynamometer; and we may then make use of the method of WEBER for estimating the intensity and duration of a current uniform while it lasts which would produce the same effects.

(39) Let $n_1$, $n_2$ be the roots of the equation

$$(LN-M^2)n^2+(RN+LS)n+RS=0, \quad \ldots \ldots \quad (16)$$

and let the primary coil be acted on by a constant electromotive force R$c$, so that $c$ is the constant current it could maintain; then the complete solution of the equations for making contact is

$$x=\frac{c}{S}\frac{n_1n_2}{n_1-n_2}\left\{\left(\frac{S}{n_1}+N\right)e^{n_1t}-\left(\frac{S}{n_2}+N\right)e^{n_2t}+S\frac{n_1-n_2}{n_1n_2}\right\}, \quad \ldots \quad (17)$$

$$y=\frac{cM}{S}\frac{n_1n_2}{n_1-n_2}\{e^{n_1t}-e^{n_2t}\}. \quad \ldots \ldots \quad (18)$$

From these we obtain for calculating the impulse on the dynamometer,

$$\int x^2dt=c^2\left\{t-\tfrac{3}{2}\frac{L}{R}-\tfrac{1}{2}\frac{M^2}{RN+LS}\right\}, \quad \ldots \ldots \quad (19)$$

$$\int y^2dt=c^2\tfrac{1}{2}\frac{M^2R}{S(RN+LS)}. \quad \ldots \ldots \quad (20)$$

The effects of the current in the secondary coil on the galvanometer and dynamometer are the same as those of a uniform current

$$-\tfrac{1}{2}c\frac{MR}{RN+LS}$$

for a time

$$2\left(\frac{L}{R}+\frac{N}{S}\right).$$

(40) The equation between work and energy may be easily verified. The work done by the electromotive force is

$$\xi\int xdt=c^2(Rt-L).$$

Work done in overcoming resistance and producing heat,

$$R\int x^2dt+S\int y^2dt=c^2(Rt-\tfrac{3}{2}L).$$

Energy remaining in the system,

$$=\tfrac{1}{2}c^2L.$$

(41) If the circuit R is suddenly and completely interrupted while carrying a current $c$, then the equation of the current in the secondary coil would be

$$y=c\frac{M}{N}e^{-\frac{S}{N}t}.$$

This current begins with a value $c\frac{M}{N}$, and gradually disappears.

The total quantity of electricity is $c\frac{\mathrm{M}}{\mathrm{S}}$, and the value of $\int y^2 dt$ is $c^2\frac{\mathrm{M}^2}{2\mathrm{SN}}$.

The effects on the galvanometer and dynamometer are equal to those of a uniform current $\frac{1}{2}c\frac{\mathrm{M}}{\mathrm{N}}$ for a time $2\frac{\mathrm{N}}{\mathrm{S}}$.

The heating effect is therefore greater than that of the current on making contact.

(42) If an electromotive force of the form $\xi=\mathrm{E}\cos pt$ acts on the circuit R, then if the circuit S is removed, the value of $x$ will be

$$x=\frac{\mathrm{E}}{\mathrm{A}}\sin(pt-\alpha),$$

where

$$\mathrm{A}^2=\mathrm{R}^2+\mathrm{L}^2p^2,$$

and

$$\tan\alpha=\frac{\mathrm{L}p}{\mathrm{R}}.$$

The effect of the presence of the circuit S in the neighbourhood is to alter the value of A and $\alpha$, to that which they would be if R become

$$\mathrm{R}+p^2\frac{\mathrm{MS}}{\mathrm{S}^2+p^2\mathrm{N}^2},$$

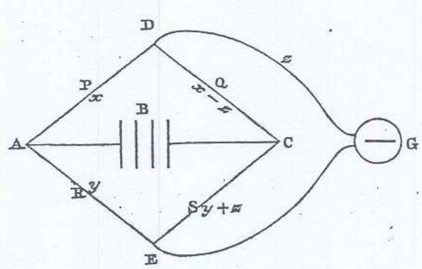
and L became

$$\mathrm{L}-p^2\frac{\mathrm{MN}}{\mathrm{S}^2+p^2\mathrm{N}^2}.$$

Hence the effect of the presence of the circuit S is to increase the apparent resistance and diminish the apparent self-induction of the circuit R.

*On the Determination of Coefficients of Induction by the Electric Balance.*

(43) The electric balance consists of six conductors joining four points, A C D E, two and two. One pair, A C, of these points is connected through the battery B. The opposite pair, D E, is connected through the galvanometer G. Then if the resistances of the four remaining conductors are represented by P, Q, R, S, and the currents in them by $x$, $x-z$, $y$, and $y+z$, the current through G will be $z$. Let the potentials at the four points be A, C, D, E. Then the conditions of steady currents may be found from the equations




$$\left.\begin{array}{ll}\mathrm{P}x=\mathrm{A}-\mathrm{D} & \mathrm{Q}(x-z)=\mathrm{D}-\mathrm{C},\\ \mathrm{R}y=\mathrm{A}-\mathrm{E} & \mathrm{S}(y+z)=\mathrm{E}-\mathrm{C},\\ \mathrm{G}z=\mathrm{D}-\mathrm{E} & \mathrm{B}(x+y)=-\mathrm{A}+\mathrm{C}+\mathrm{F}.\end{array}\right\} \quad \ldots \ldots \quad (21)$$

Solving these equations for $z$, we find

$$z\left\{\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{Q}}+\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{S}}+\mathrm{B}\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{R}}\right)\left(\frac{1}{\mathrm{Q}}+\frac{1}{\mathrm{S}}\right)+\mathrm{G}\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{Q}}\right)\left(\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{S}}\right)+\frac{\mathrm{BG}}{\mathrm{PQRS}}(\mathrm{P}+\mathrm{Q}+\mathrm{R}+\mathrm{S})\right\}=\mathrm{F}\left(\frac{1}{\mathrm{PS}}-\frac{1}{\mathrm{QR}}\right). \quad (22)$$

In this expression F is the electromotive force of the battery, $z$ the current through the galvanometer when it has become steady. P, Q, R, S the resistances in the four arms. B that of the battery and electrodes, and G that of the galvanometer.

(44) If PS=QR, then $z=0$, and there will be no steady current, but a transient current through the galvanometer may be produced on making or breaking circuit on account of induction, and the indications of the galvanometer may be used to determine the coefficients of induction, provided we understand the actions which take place.

We shall suppose PS=QR, so that the current $z$ vanishes when sufficient time is allowed, and

$$x(\mathrm{P}+\mathrm{Q})=y(\mathrm{R}+\mathrm{S})=\frac{\mathrm{F}(\mathrm{P}+\mathrm{Q})(\mathrm{R}+\mathrm{S})}{(\mathrm{P}+\mathrm{Q})(\mathrm{R}+\mathrm{S})+\mathrm{B}(\mathrm{P}+\mathrm{Q})(\mathrm{R}+\mathrm{S})}.$$

Let the induction coefficients between P, Q, R S, be given by the following Table, the coefficient of induction of P on itself being $p$, between P and Q, $h$, and so on.

| | P | Q | R | S |
|---|---|---|---|---|
| P | $p$ | $h$ | $k$ | $l$ |
| Q | $h$ | $q$ | $m$ | $n$ |
| R | $k$ | $m$ | $r$ | $o$ |
| S | $l$ | $n$ | $o$ | $s$ |

Let $g$ be the coefficient of induction of the galvanometer on itself, and let it be out of the reach of the inductive influence of P, Q, R, S (as it must be in order to avoid direct action of P, Q, R, S on the needle). Let X, Y, Z be the integrals of $x$, $y$, $z$ with respect to $t$. At making contact $x$, $y$, $z$ are zero. After a time $z$ disappears, and $x$ and $y$ reach constant values. The equations for each conductor will therefore be

$$\left.\begin{aligned}&\mathrm{PX}+(p+h)x+(k+l)y=\int\mathrm{A}dt-\int\mathrm{D}dt,\\&\mathrm{Q}(\mathrm{X}-\mathrm{Z})+(h+q)x+(m+n)y=\int\mathrm{D}dt-\int\mathrm{C}dt,\\&\mathrm{RY}+(k+m)x+(r+o)y=\int\mathrm{A}dt-\int\mathrm{E}dt,\\&\mathrm{S}(\mathrm{Y}+\mathrm{Z})+(l+n)x+(o+s)y=\int\mathrm{E}dt-\int\mathrm{C}dt,\\&\mathrm{GZ}=\int\mathrm{D}td-\int\mathrm{E}dt.\end{aligned}\right\}\quad\ldots\ldots\quad(24)$$

Solving these equations for Z, we find

$$\left.\begin{aligned}&\mathrm{Z}\left\{\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{Q}}+\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{S}}+\mathrm{B}\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{R}}\right)\left(\frac{1}{\mathrm{Q}}+\frac{1}{\mathrm{S}}\right)+\mathrm{G}\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{Q}}\right)\left(\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{S}}\right)+\frac{\mathrm{BG}}{\mathrm{PQRS}}(\mathrm{P}+\mathrm{Q}+\mathrm{R}+\mathrm{S})\right\}\\&\quad=-\mathrm{F}\frac{1}{\mathrm{PS}}\left\{\frac{p}{\mathrm{P}}-\frac{q}{\mathrm{Q}}-\frac{r}{\mathrm{R}}+\frac{s}{\mathrm{S}}+h\left(\frac{1}{\mathrm{P}}-\frac{1}{\mathrm{Q}}\right)+k\left(\frac{1}{\mathrm{R}}-\frac{1}{\mathrm{P}}\right)+l\left(\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{Q}}\right)-m\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{S}}\right)\right.\\&\quad\left.+n\left(\frac{1}{\mathrm{Q}}-\frac{1}{\mathrm{S}}\right)+o\left(\frac{1}{\mathrm{S}}-\frac{1}{\mathrm{R}}\right)\right\}.\end{aligned}\right\}\quad(25)$$

(45) Now let the deflection of the galvanometer by the instantaneous current whose intensity is Z be $\alpha$.

Let the permanent deflection produced by making the ratio of PS to QR, $\varrho$ instead of unity, be $\theta$.

Also let the time of vibration of the galvanometer needle from rest to rest be T.

Then calling the quantity

$$\frac{p}{\mathrm{P}}-\frac{q}{\mathrm{Q}}-\frac{r}{\mathrm{R}}+\frac{s}{\mathrm{S}}+h\left(\frac{1}{\mathrm{P}}-\frac{1}{\mathrm{Q}}\right)+k\left(\frac{1}{\mathrm{R}}-\frac{1}{\mathrm{P}}\right)+l\left(\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{Q}}\right)-m\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{S}}\right)+n\left(\frac{1}{\mathrm{Q}}-\frac{1}{\mathrm{S}}\right)+o\left(\frac{1}{\mathrm{S}}-\frac{1}{\mathrm{R}}\right)=\tau, \quad (26)$$

we find

$$\frac{\mathrm{Z}}{z}=\frac{2\sin\frac{1}{2}\alpha}{\tan\theta}\frac{\mathrm{T}}{\pi}=\frac{\tau}{1-\varrho}. \quad \ldots \quad (27)$$

In determining $\tau$ by experiment, it is best to make the alteration of resistance in one of the arms by means of the arrangement described by Mr. JENKIN in the Report of the British Association for 1863, by which any value of $\varrho$ from 1 to 1·01 can be accurately measured.

We observe $(\alpha)$ the greatest deflection due to the impulse of induction when the galvanometer is in circuit, when the connexions are made, and when the resistances are so adjusted as to give no permanent current.

We then observe $(\beta)$ the greatest deflection produced by the permanent current when the resistance of one of the arms is increased in the ratio of 1 to $\varrho$, the galvanometer not being in circuit till a little while after the connexion is made with the battery.

In order to eliminate the effects of resistance of the air, it is best to vary $\varrho$ till $\beta=2\alpha$ nearly; then

$$\tau=\mathrm{T}\frac{1}{\pi}(1-\varrho)\frac{2\sin\frac{1}{2}\alpha}{\tan\frac{1}{2}\beta}. \quad \ldots \quad (28)$$

If all the arms of the balance except P consist of resistance coils of very fine wire of no great length and doubled before being coiled, the induction coefficients belonging to these coils will be insensible, and $\tau$ will be reduced to $\frac{p}{\mathrm{P}}$. The electric balance therefore affords the means of measuring the self-induction of any circuit whose resistance is known.

(46) It may also be used to determine the coefficient of induction between two circuits, as for instance, that between P and S which we have called $m$; but it would be more convenient to measure this by directly measuring the current, as in (37), without using the balance. We may also ascertain the equality of $\frac{p}{\mathrm{P}}$ and $\frac{q}{\mathrm{Q}}$ by there being no current of induction, and thus, when we know the value of $p$, we may determine that of $q$ by a more perfect method than the comparison of deflections.

### *Exploration of the Electromagnetic Field.*

(47) Let us now suppose the primary circuit A to be of invariable form, and let us explore the electromagnetic field by means of the secondary circuit B, which we shall suppose to be variable in form and position.

We may begin by supposing B to consist of a short straight conductor with its extremities sliding on two parallel conducting rails, which are put in connexion at some distance from the sliding-piece.

Then, if sliding the moveable conductor in a given direction increases the value of M, a negative electromotive force will act in the circuit B, tending to produce a negative current in B during the motion of the sliding-piece.

If a current be kept up in the circuit B, then the sliding-piece will itself tend to move in that direction, which causes M to increase. At every point of the field there will always be a certain direction such that a conductor moved in that direction does not experience any electromotive force in whatever direction its extremities are turned. A conductor carrying a current will experience no mechanical force urging it in that direction or the opposite.

This direction is called the direction of the line of magnetic force through that point.

Motion of a conductor across such a line produces electromotive force in a direction perpendicular to the line and to the direction of motion, and a conductor carrying a current is urged in a direction perpendicular to the line and to the direction of the current.

(48) We may next suppose B to consist of a very small plane circuit capable of being placed in any position and of having its plane turned in any direction. The value of M will be greatest when the plane of the circuit is perpendicular to the line of magnetic force. Hence if a current is maintained in B it will tend to set itself in this position, and will of itself indicate, like a magnet, the direction of the magnetic force.

*On Lines of Magnetic Force.*

(49) Let any surface be drawn, cutting the lines of magnetic force, and on this surface let any system of lines be drawn at small intervals, so as to lie side by side without cutting each other. Next, let any line be drawn on the surface cutting all these lines, and let a second line be drawn near it, its distance from the first being such that the value of M for each of the small spaces enclosed between these two lines and the lines of the first system is equal to unity.

In this way let more lines be drawn so as to form a second system, so that the value of M for every reticulation formed by the intersection of the two systems of lines is unity.

Finally, from every point of intersection of these reticulations let a line be drawn through the field, always coinciding in direction with the direction of magnetic force.

(50) In this way the whole field will be filled with lines of magnetic force at regular intervals, and the properties of the electromagnetic field will be completely expressed by them.

For, 1st, If any closed curve be drawn in the field, the value of M for that curve will be expressed by the *number* of lines of force which *pass through* that closed curve.

2ndly. If this curve be a conducting circuit and be moved through the field, an electromotive force will act in it, represented by the rate of decrease of the number of lines passing through the curve.

3rdly. If a current be maintained in the circuit, the conductor will be acted on by forces tending to move it so as to increase the number of lines passing through it, and