

The value of L for this coil was found in the following way.

The value of L was calculated by the preceding formula for six different cases, in which the rectangular section considered has always the same breadth, while the depth was

$$A, B, C, A+B, B+C, A+B+C,$$

and $n=1$ in each case.

Calling the results

$$L(A), L(B), L(C), \&c.,$$

we calculate the coefficient of mutual induction $M(AC)$ of the two coils thus,

$$2ACM(AC) = (A+B+C)^2L(A+B+C) - (A+B)^2L(A+B) - (B+C)^2L(B+C) + B^2L(B).$$

Then if n_1 is the number of windings in the coil A and n_2 in the coil B, the coefficient of self-induction of the two coils together is

$$L = n_1^2L(A) + 2n_1n_2L(AC) + n_2^2L(B).$$

(114) These values of L are calculated on the supposition that the windings of the wire are evenly distributed so as to fill up exactly the whole section. This, however, is not the case, as the wire is generally circular and covered with insulating material. Hence the current in the wire is more concentrated than it would have been if it had been distributed uniformly over the section, and the currents in the neighbouring wires do not act on it exactly as such a uniform current would do.

The corrections arising from these considerations may be expressed as numerical quantities, by which we must multiply the length of the wire, and they are the same whatever be the form of the coil.

Let the distance between each wire and the next, on the supposition that they are arranged in square order, be D, and let the diameter of the wire be d , then the correction for diameter of wire is

$$+2 \left(\log \frac{D}{d} + \frac{4}{3} \log 2 + \frac{\pi}{3} - \frac{11}{6} \right).$$

The correction for the eight nearest wires is

$$+0.0236.$$

For the sixteen in the next row

$$+0.00083.$$

These corrections being multiplied by the length of wire and added to the former result, give the true value of L, considered as the measure of the potential of the coil on itself for unit current in the wire when that current has been established for some time, and is uniformly distributed through the section of the wire.

(115) But at the commencement of a current and during its variation the current is not uniform throughout the section of the wire, because the inductive action between different portions of the current tends to make the current stronger at one part of the section than at another. When a uniform electromotive force P arising from any cause

acts on a cylindrical wire of specific resistance ρ , we have

$$p\rho = P - \frac{dF}{dt},$$

where F is got from the equation

$$\frac{d^2F}{dr^2} + \frac{1}{r} \frac{dF}{dr} = -4\pi\mu p,$$

r being the distance from the axis of the cylinder.

Let one term of the value of F be of the form Tr^n , where T is a function of the time, then the term of p which produced it is of the form

$$-\frac{1}{4\pi\mu} n^2 T r^{n-2}.$$

Hence if we write

$$F = T + \frac{\mu\pi}{\rho} \left(-P + \frac{dT}{dt} \right) r^2 + \frac{\mu\pi^2}{\rho} \frac{1}{1^2 \cdot 2^2} \frac{dT^2}{dt^2} r^4 + \&c.$$

$$p\rho = \left(P + \frac{dT}{dt} \right) - \frac{\mu\pi}{\rho} \frac{d^2T}{dt^2} r^2 - \frac{\mu\pi^2}{\rho} \frac{1}{1^2 \cdot 2^2} \frac{d^3T}{dt^3} r^4 - \&c.$$

The total counter current of self-induction at any point is

$$\int \left(\frac{P}{\rho} - p \right) dt = \frac{1}{\rho} T + \frac{\mu\pi}{\rho^2} \frac{dT}{dt} r^2 + \frac{\mu^2\pi^2}{\rho^3} \frac{1}{1^2 \cdot 2^2} \frac{d^2T}{dt^2} r^4 + \&c.$$

from $t=0$ to $t=\infty$.

$$\text{When } t=0, p=0, \quad \therefore \left(\frac{dT}{dt} \right)_0 = P, \quad \left(\frac{d^2T}{dt^2} \right)_0 = 0, \quad \&c.$$

$$\text{When } t=\infty, p = \frac{P}{\rho}, \quad \therefore \left(\frac{dT}{dt} \right)_\infty = 0, \quad \left(\frac{d^2T}{dt^2} \right)_\infty = 0, \quad \&c.$$

$$\int_0^\infty \int_0^r 2\pi \left(\frac{P}{\rho} - p \right) r dr dt = \frac{1}{\rho} T \pi r^2 + \frac{1}{2} \frac{\mu\pi^2}{\rho^2} \frac{dT}{dt} r^4 + \frac{\mu^2\pi^3}{\rho^3} \frac{1}{1^2 \cdot 2^2 \cdot 3} \frac{d^2T}{dt^2} r^6 + \&c.$$

from $t=0$ to $=\infty$.

$$\text{When } t=0, p=0 \text{ throughout the section, } \therefore \left(\frac{dT}{dt} \right)_0 = P, \quad \left(\frac{d^2T}{dt^2} \right)_0 = 0, \quad \&c.$$

$$\text{When } t=\infty, p=0 \text{ throughout } \dots \therefore \left(\frac{dT}{dt} \right)_\infty = 0, \quad \left(\frac{d^2T}{dt^2} \right)_\infty = 0, \quad \&c.$$

Also if l be the length of the wire, and R its resistance,

$$R = \frac{\rho l}{\pi r^2};$$

and if C be the current when established in the wire, $C = \frac{Pl}{R}$.

The total counter current may be written

$$\frac{l}{R} (T_\infty - T_0) - \frac{1}{2} \mu \frac{l}{R} C = -\frac{LC}{R} \text{ by } \S (35).$$

Now if the current instead of being variable from the centre to the circumference of the section of the wire had been the same throughout, the value of F would have been

$$F = T + \mu\gamma \left(1 - \frac{r^2}{r_0^2}\right),$$

where γ is the current in the wire at any instant, and the total countercurrent would have been

$$\int_0^\infty \int_0^r \frac{1}{g} \frac{dF}{dt} 2\pi r dr = \frac{l}{R} (T_\infty - T_0) - \frac{3}{4} \mu \frac{l}{R} C = -\frac{L'C}{R}, \text{ say.}$$

Hence

$$L = L' - \frac{1}{4} \mu l,$$

or the value of L which must be used in calculating the self-induction of a wire for variable currents is less than that which is deduced from the supposition of the current being constant throughout the section of the wire by $\frac{1}{4} \mu l$, where l is the length of the wire, and μ is the coefficient of magnetic induction for the substance of the wire.

(116) The dimensions of the coil used by the Committee of the British Association in their experiments at King's College in 1864 were as follows:—

Mean radius	= $a =$.158194	metre.
Depth of each coil	= $b =$.01608	
Breadth of each coil	= $c =$.01841	
Distance between the coils	= .02010	
Number of windings	$n =$ 313	
Diameter of wire	= .00126	

The value of L derived from the first term of the expression is 437440 metres.

The correction depending on the radius not being infinitely great compared with the section of the coil as found from the second term is -7345 metres.

The correction depending on the diameter of the wire is	} + .44997
per unit of length	
Correction of eight neighbouring wires	+ .0236
For sixteen wires next to these	+ .0008
Correction for variation of current in different parts of section	- .2500
Total correction per unit of length22437
Length	311.236 metres.
Sum of corrections of this kind	70 "
Final value of L by calculation	430165 "

This value of L was employed in reducing the observations, according to the method explained in the Report of the Committee*. The correction depending on L varies as the square of the velocity. The results of sixteen experiments to which this correction had been applied, and in which the velocity varied from 100 revolutions in seventeen seconds to 100 in seventy-seven seconds, were compared by the method of

* British Association Reports, 1863, p. 169.

least squares to determine what further correction depending on the square of the velocity should be applied to make the outstanding errors a minimum.

The result of this examination showed that the calculated value of L should be multiplied by 1.0618 to obtain the value of L, which would give the most consistent results.

We have therefore L by calculation	430165 metres.
Probable value of L by method of least squares	456748 „
Result of rough experiment with the Electric Balance (see § 46)	410000 „

The value of L calculated from the dimensions of the coil is probably much more accurate than either of the other determinations.