Vacuum Energy

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Comments:

A comprehensive review of Vacuum Energy, which is an extended version of a poster presented at Lüderitz (2000). This is not a review of the cosmological constant *per se*, but rather vacuum energy in general, my approach to the cosmological constant is not standard. Lots of very small changes and several additions for the second and third versions: constructive feedback still welcome, but the next version will be sometime in coming due to my sporadiac internet access.

First Version 153 pages, 368 references.

Second Version 161 pages, 399 references.

Third Version 167 pages, 412 references.

Abstract

There appears to be *three*, perhaps related, ways of approaching the nature of vacuum energy. The *first* is to say that it is just the lowest energy state of a given, usually quantum, system. The *second* is to equate vacuum energy with the Casimir energy. The *third* is to note that an energy difference from a complete vacuum might have some long range effect, typically this energy difference is interpreted as the cosmological constant. All three approaches are reviewed, with an emphasis on recent work. It is hoped that this review is comprehensive in scope. There is a discussion on whether there is a relation between vacuum energy and inertia. The solution suggested here to the nature of the vacuum is that Casimir energy can produce short range effects because of boundary conditions, but that at long range there is no overall effect of vacuum energy, unless one considers lagrangians of higher order than Einstein's as vacuum induced. No original calculations are presented in support of this position.

Contents

1 Introduction.

1.1 Forward.

In physics there are a great variety of views of what vacuum energy is. It is understood differently in many formalisms and domains of study, and ideas from one area are sometimes used in another; for example a microscopic origin for the cosmological constant is often used to justify its inclusion in cosmology. A *narrow* view is that vacuum energy is synonymous with the cosmological constant; why this is the case is explained in §2.1¶2. A *wide* view is that vacuum energy is just the lowest energy of any system under consideration. Finkelstein [125] (1991) takes an *extreme* view of what the vacuum is:

> The structure of the vacuum is the central problem of physics today: the fusion of the theories of gravity and the quantum is a subproblem.

In general relativity gravitational energy is hard to calculate, and often ambiguous, especially in non-asymptotically flat spacetimes. Now how vacuum energy fits into gravitation is not straightforward as the gravitational field might have its own vacuum energy, this is discussed in §6. Einstein [111] (1924) discusses the effect of "ether", perhaps the ether can be thought of as vacuum energy. Vacuum energy, like any other energy, can be thought of as having an equivalent mass via $E = mc^2$. Moving masses have inertia, and this is one way of looking at how inertia is related to vacuum energy, this is discussed in §7. The principle of equivalence, §5, can be formulated in a way which says that the laws of physics are the same in all inertial frames. An inertial frame is thus a primitive concept, and one way of thinking of it is of it occurring because of some property of the vacuum.

1.2 Historical Background.

The presentation here of the history of zero-point energy largely follows Sciama [310] (1991), the is also a historical discussion in Lima and Maia [391] (1995).

1.2.1 The Zero-point Energy of a Harmonic Oscillator.

Zero-point energy was introduced into physics by Max Planck in 1911. In a renewed attempt to understand the interaction between matter and radiation and its relationship to the black body spectrum Planck put forward the hypothesis that the absorption of radiation occurs continuously while its emission is discrete and in energy units of $h\nu$, On this hypothesis the average energy $\bar{\varepsilon}$ of a harmonic oscillator at temperature T would be given by

$$\bar{\varepsilon} = \frac{1}{2}h\nu + B(\nu, T) \tag{1.1}$$

where

$$B(\nu, T) \equiv \frac{h\nu}{e^{h\nu/kT} - 1}. \tag{1.2}$$

Thus the oscillator would have a finite zero-point energy $\frac{1}{2}h\nu$ even at the absolute zero of temperature. When it comes to quantum field theory, there can be thought of as being a quantum oscillation at each point in spacetime, this leads to the zero-point energy giving an overall infinite energy contribution, so that there is a problem in how to adjust this contribution in order to allow quantum field theory to be well defined.

Planck abandoned his discrete emission of energy, in units of $h\nu$, hypothesis in 1914 when Fokker showed that an assembly of rotating dipoles interacting classically with electromagnetic radiation would posses statistical properties (such as specific heat) in conflict with observation. Planck then became convinced that no classical discussion could lead in a satisfactory manner to a derivation of his distribution for black body radiation, in other words the distribution was fundamental and h could not be derived from previously know properties. Nevertheless according to Sciama [310] (1991) the idea of zero-point motion for a quantum harmonic oscillator continued to intrigue physicists, and the possibility was much discussed before its existence was definitely shown in 1925 to be required by quantum mechanics, as a direct consequence of Heisenberg's uncertainty principle.

The Einstein-Hopf (1910) derivation of the black body radiation spectrum, requires an expression for zero-point motion. That derivation involved a study of the interchange of energy and momentum between a harmonic oscillator and the radiation field. It led to the Rayleigh-Jeans distribution rather than to Planck's because of the classical assumptions which were originally used for the harmonic oscillators and the radiation field. In the Einstein-Hopf calculation the mean square momentum of an oscillator was found to be proportional to the mean energy of the oscillator and to the mean energy density of the radiation field. Einstein and Stern now included the zero-point energy of the oscillator in its mean energy, in the hope of deriving the Planck rather than the Rayleigh-Jeans distribution, but found that they could do so only if they took $h\nu$ rather than $\frac{1}{2}h\nu$ for the zero-point energy.

According to Sciama [310] (1991) the reason for this difficulty is their neglect of the zero-point energy of the radiation field itself. It is not a consistent procedure to link together two physical systems only one of which possesses zero-point fluctuations in a steady state. These fluctuations would simply drive zero-point fluctuations in the other system or be damped out, depending on the relative number of degrees of freedom in the two systems. Nerst (1916), first proposed that 'empty' space was everywhere filled with zero-point electromagnetic radiation. According to Sciama [310] (1991) the main considerations of Einstein and Stern are wrong; when one takes the classical limit $kT >> h\nu$ for the Planck distribution, one finds

$$B(\nu, T) \to kT - \frac{1}{2}h\nu + h\nu \times O\left(\frac{h\nu}{kT}\right), \tag{1.3}$$

whereas

$$\frac{1}{2}h\nu + B(\nu, T) \to kT + h\nu \times O\left(\frac{h\nu}{kT}\right). \tag{1.4}$$

Thus the correct classical limit is obtained only if the zero-point energy is included.

Einstein and Stern gave two independent arguments in favour of retaining the zero-point energy, the *first* involved rotational specific heats of molecular gases and the *second* from an attempted, derivation of the Planck distribution itself. In the first argument it was assumed that a freely rotating molecule would possess a zero-point energy, this assumption is now known not to be compatible with quantum mechanics.

Nogueira and Maia [258] (1995) discuss the possibility that there might be no zero-point energy.

1.2.2 The Experimental Verification of Zero-point Energy.

For a single harmonic oscillator, the existence of a zero-point energy would not change the spacing between the various oscillator levels and so would not show up in the energy spectrum. It might, of course, alter the gravitational field produced by the oscillator. For the experimental physicists influenced by Planck and also by Einstein and Stern, demonstrating the existence of zero-point energy amounted to finding a system in which the *difference* in this energy for different parts of the system could be measured. According to Sciama [310] (1991) it was soon realized that a convenient way to do this was to look for isotope effects in the vibrational spectra of molecules. The small change in mass associated with an isotopic replacement would lead to a small change in the zero-point energy, and in the energies of all vibrational levels. These changes might then show up in the vibrational spectrum of the system.

Many attempts were made to find this effect, but the first conclusive experiment was made by Mulliken in 1925, in his studies of the band spectrum of boron monoxide. This demonstration, made only a few months before Heisenberg (1925) first derived the zero-point energy for a harmonic oscillator from his new matrix mechanics, provided the first experimental verification of the new quantum theory. New quantum theory here meaning Heisenberg-Schrödinger theory as opposed to the old Bohr-Sommerfeld quantum theory.

Since then, zero-point effects have become commonplace in quantum physics, **three** examples are in spectroscopy, in chemical reactions, and in solid-state physics. Perhaps the most dramatic example is their role in maintaining helium in the liquid state under its own vapour pressure at (almost) absolute zero. The zero-point motion of the atoms keep them sufficiently far apart on average so that the attractive forces between them are too weak to cause solidification. This can be expressed in a rough way by defining an effective temperature T_{eff} where kT_{eff} is equal to the zero-point energy per atom. For helium T_{eff} exceeds a classical estimate of the melting point under its own vapour pressure. Thus, even close to absolute zero helium is 'hot' enough to be liquid.

1.3 Finding, Creating and Exploiting Vacua.

Perhaps related to the nature of vacuum energy is the nature of vacua in general. Absolute vacua cannot be *created* or *found*. Looking at "found" *first*, note that physical spacetime is never a vacuum, for example interplanetary space is not a vacuum, but has density $\rho = 10^{-29}$g.cm.$^{-3}$ or 10^{-5} protons cm.$^{-3}$, the magnitude of the upper limit of the effective cosmological constant is about $\rho_\Lambda = 10^{-16}$g.cm.$^{-3}$, see Roberts §5.2 [296] (1998). The Universe itself is not a vacuum but has various densities associated with itself, see §4 below. A particle gas, as used in kinetic theory, can be thought of as billiard balls moving through space, the space being a vacuum: however such a theory is not fundamental, whereas field theories are, and in them particles moving through space are replaced by fields defined at each point, if the fields are non-zero there is no vacuum. To look at "created" *second*, note that a particle accelerator needs to work in as close to a vacuum as can be achieved. A recent discussion of a particular approach to vacuum creation for particle accelerators is Collins *et al* [85] (2000). They study ion induced vacuum instability which was first observed in the Intersecting Proton Storage Rings (ISR) at CERN and in spite of substantial vacuum improvements, this instability remains a limitation of the maximum beam current throughout the operation of the machine. Extensive laboratory studies and dedicated machine experiments were made during this period to understand the details of this effect and to identify ways of increasing the limit to higher beam currents. Stimulated by the recent design work for the LHC vacuum system, the interest in this problem has been revived with a new critical review of the parameters which determine the pressure run-away in a given vacuum system with high intensity beams.

Given the nature of the quantum vacuum and its fluctuations, it can be imagined that there is a lot of energy out there floating around in the vacuum: is there any way to get hold of it? The *second law of thermodynamics* might suggest not, but that law is not cognizant of quantum field theory (nor of gravity).

There are occasional discussions on whether vacuum energy can be *exploited*, typically used to power electrical or mechanical devices, an example is Yam [361] (1997) who concludes that zero-point energy probably cannot be tapped, see also §3.7 where the exploitation of Casimir energy is discussed. Xue [399] (2000) presents and studies a possible mechanism of extracting energies from the vacuum by external classical fields. Taking a constant magnetic field as an example, he discusses why and how the vacuum energy can be released in the context of quantum field theories. In addition, he gives a theoretical computation showing how much vacuum energies can be released. He discusses the possibilities of experimentally detecting such a vacuum-energy releasing. Scandurra [410] (2001) extends the fundamental laws of thermodynamics and of the concept of entropy to the ground state fluctuations of quantum fields. He critically analyzes a device to extract energy from the vacuum. He finds that no energy can be extracted cyclically from the vacuum.

There are *three things* suggest a positive possibility of energy extraction. *Firstly*, the theory of quantum evaporation from "black holes" can be thought of as a form of energy extraction from the quantum vacuum. *Secondly*, the inflationary universe can also be thought of as extracting energy from the vacuum, see §4.4. So there is the possibility that what is happening in either of those cases could be made to happen in a controlled way in a laboratory. The *third* things is an analogy: it is said in physics text books that you can not extract the vast amount of thermal energy in the sea, because there is no lower temperature heat bath available; so it is unavailable energy, despite there being so much of it there. Ellis [113] (1979) pointed out that in fact it can be extracted when you remember that the dark night sky acts as a heat sink at 3K (the temperature of the Cosmic background radiation). In some ways this seems reminiscent of the energy bath that is the quantum vacuum. If some particles in that vacuum have an effective energy of greater than 3K, maybe one can radiate them off to the night sky and leave their partner behind.

2 The Quantum Field Theory Vacuum.

2.1 The Vacuum State of QFT's in general.

From equation 1.1 at zero temperature an harmonic oscillator contributes $\frac{1}{2}h\nu$ to the energy. Quantum field theory (QFT) assumes that each point of a field can be treated as a quantum system, typically as a quantum harmonic oscillator, thus each point contributes $\frac{1}{2}h\nu$ to the energy resulting in an overall infinite energy. Various ways have been contrived to deal with this infinite overall energy, *two* examples are: *firstly* renormalization schemes which invoke the idea that it is only energy differences that are measurable and that attempts to subtract infinite energy from quantum vacuum energy, see §2.16 below, and *secondly* supersymmetry where the infinite vacuum energy of one field is cancelled out term by term by its supersymmetric partner's opposite sign vacuum energy. see §2.6 below. Dmitriyev [401] (1992) describes elastic analogs of the vacuum. Fields are used to describe matter and the relation of the vacuum to elementary matter is discussed in Stöcker *et al* [328] (1997).

Fauser [402] (1997) proposes a geometric method to parameterize inequivalent vacua by dynamical data. Introducing quantum Clifford algebras with arbitrary bilinear forms we distinguish isomorphic algebras –as Clifford algebras– by different filtrations resp. induced gradings. The idea of a vacuum is introduced as the unique algebraic projection on the base field embedded in the Clifford algebra, which is however equivalent to the term vacuum in axiomatic quantum field theory and the GNS construction in C^*-algebras. This approach is shown to be equivalent to the usual picture which fixes one product but employs a variety of GNS states. The most striking novelty of the geometric approach is the fact that dynamical data fix uniquely the vacuum and that positivity is not required. The usual concept of a statistical quantum state can be generalized to geometric meaningful but non-statistical, non-definite, situations. Furthermore, an algebraization of states takes place. An application to physics is provided by an U(2)-symmetry producing a gap-equation which governs a phase transition. The parameterization of all vacua is explicitly calculated from propagator matrix elements. A discussion of the relation to BCS theory and Bogoliubov-Valatin transformations is given.

Chen [400] (2001) provides a new explaination of the meaning of negative energies in the relativistic theory. On the basis of that he presents two new conjectures. According to the conjectures, particles have two sorts of existing forms which are symmetric. From this he presents a new Lagrangian density and a new quantization method for QED. That the energy of the vacuum state is equal to zero is naturally obtained. From this he can easily determine the cosmological constant according to experiments, and it is possible to correct nonperturbational methods which depend on the energy of the ground state in quantum field theory.

Grandpeix & Lurcat [404] (2001) first sketch the "frame problem": the motion of an isolated particle obeys a simple law in galilean frames, but how does the galilean character of the frame manifest itself at the place of the particle? A description of vacuum as a system of virtual particles will help to answer this question. For future application to such a description, they define and study the notion of global particle. To this end, a systematic use of the Fourier transformation on the Poincare group is needed. The state of a system of n free particles is represented by a statistical operator W, which defines an operator-valued measure on the $n-th$power of the dual of the Poincare group. The inverse Fourier-Stieltjes transform of that measure is called the characteristic function of the system; it is a function on the $n-th$ power of the Poincare group. The main notion is that of global characteristic function: it is the restriction of the characteristic function to the diagonal subgroup ; it represents the state of the system, considered as a single particle. The main properties of characteristic functions, and particularly of global characteristic functions, are studied. A mathematical Appendix defines two functional spaces involved. They describe vacuum as a system of virtual particles, some of which have negative energies. Any system of vacuum particles is a part of a keneme, i.e. of a system of n

particles which can, without violating the conservation laws, annihilate in the strict sense of the word (transform into nothing). A keneme is a homogeneous system, i.e. its state is invariant by all transformations of the invariance group. But a homogeneous system is not necessarily a keneme. In the simple case of a spin system, where the invariance group is $SU(2)$, a homogeneous system is a system whose total spin is unpolarized; a keneme is a system whose total spin is zero. The state of a homogeneous system is described by a statistical operator with infinite trace (von Neumann), to which corresponds a characteristic distribution. The characteristic distributions of the homogeneous systems of vacuum are defined and studied. Finally they show how this description of vacuum can be used to solve the frame problem posed in the first paper.

The standard argument that a non-zero vacuum leads to a cosmological constant is as follows. Consider the action of a single scalar field with potential $V(\phi)$ is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{ab}(\partial_a\phi)(\partial_b\phi) - V(\phi)\right] \tag{2.1}$$

where the lagrangian is

$$\mathcal{L} = -\frac{1}{2}\phi_a\phi^a + V(\phi), \tag{2.2}$$

and a typical potential, the ϕ^4 potential is

$$V(\phi) = m\phi^2 + \alpha\phi^3 + \lambda\phi^4. \tag{2.3}$$

Metric variation gives the stress

$$T_{ab} = \partial_a\phi\partial_b\phi + \frac{1}{2}g_{ab}\left[g^{cd}\partial_c\phi\partial_d\phi - V(\phi)\right], \tag{2.4}$$

if the derivatives of the scalar field vanish for a "vacuum state" $\phi_{vac} =< 0|\phi|0 >$ then

$$T^{vac}_{ab} = -\frac{1}{2}V(\phi_{vac})g_{ab} = -\rho_{vac}g_{ab}, \tag{2.5}$$

and then assuming that the stress is of the perfect fluid form

$$T_{ab} = (\rho + p)U_aU_b + p\ g_{ab}, \tag{2.6}$$

gives the pressure

$$-p_{vac} = \rho_{vac} = \frac{\Lambda}{8\pi G}. \tag{2.7}$$

This can be thought of as the vacuum expectation value of a field giving a cosmological constant contribution to the field equations, compare §4.1; this sort of contribution might produce inflation, compare §4.2. There are at least *three* problems with this approach to the vacuum. The *first* is that the assumption $\partial_a\phi = 0$ is odd, especially if one wants to argue that inflation is due to vacuum expectation of a scalar field; in an inherently non-static situation the assumption that $\partial_t\phi = 0$ is unlikely to hold. The *second* is that $V(\phi)$ is position dependent because ϕ is position dependent so $\partial_a\phi \neq 0$; so that $\phi =$ *a constant* does not hold as required for it to be a cosmological constant. Of course it can be still represented by a perfect fluid with $p = \rho$. The *third* is that I do not think that scalar fields, at least of the above form, are fundamental. Perfect fluids form a gauge system of scalar fields and I believe that all fundamental fields are gauge fields; how symmetry breaking can then be achieved using perfect fluids is discussed in Roberts [295] (1997) and §2.10 below.

An alternative approach to vacuum energy is given in Roberts [283] (1984); there a quantum-mechanical plausibility argument is introduced which suggests that matter always has a minuscule scalar field associated with itself, perhaps this can be thought of as there always being a non-zero vacuum expectation value in the form of a scalar field; and this is relate to quintessence, see §4.3. When considering quantum field theory on a curved spacetime it is usually assumed that the field equations can be used in the form $G_{ab} = 8\pi < T_{ab} >$. Hawking [166] (1975) has shown that this gives rise to an uncertainty in the local energy density of the order B^2, where $B = \text{l.u.b.}|R_{abcd}|$. This local uncertainty would necessitate a global modification of the field equations to the form $G_{ab} = 8\pi < T_{ab} > +U$, where U is an extremely small term which perturbs the equations from there classical value. Here justification is given for relating this with the splitting of T_{ab} into a matter term and a massless scalar term.

In Roberts [283] (1984) the original form of the local uncertainty of local energy density due to Hawking is not used, but instead a later variant derived by Háj'iček [163] (1977). Briefly, in curved space-time there is a difficulty in defining positive frequency, but expanding g_{ab} to second order $g_{ab} = \eta_{ab} - \frac{1}{3}R_{asbt}x^s x^t$, now defining $R = \max_{a,b,s,t=0,1,2,3} R_{asbt}$, $d = \text{l.u.b.}x^a(q)$, one sees that the corrections to the flat metric η_{ab} are of order $(d/R)^2$ and are small if $d << R$, so positive frequency can be used, after considering what states of a field can be localized in a region with radius d; Hajicak claims the the following uncertainty relations hold:

$$\frac{\Delta E}{E} \sim \frac{\hbar c}{Ed} + \frac{d^2}{R^2}, \tag{2.8}$$

so that

$$\Delta E \sim \frac{\hbar c}{d} + \frac{d^2 E}{R^2}, \tag{2.9}$$

and we also have

$$\Delta G_{00} + G_{00} = 8\pi(T_{00} + \Delta T_{00}). \tag{2.10}$$

Provided $d << 10^{-33}$cm, where quantum gravity would be expected to dominate, see for example Hawking [166] (1975), one can assume that the geometry is given by the right-hand side, *i.e.* we can put $\Delta G_{00} = 0$. Therefore

$$G_{00} = 8\pi(T_{00} + \Delta T_{00}) = 8\pi \left(T_{00} + \frac{\hbar c}{d} + \frac{d^2 T_{00}}{R^2} \right). \tag{2.11}$$

For regions with significant curvature $\hbar c/(Ed) < d^2/R^2$ so that

$$G_{00} = 8\pi T_{00}(1 + d^2/R^2). \tag{2.12}$$

Substituting in values for E the energy from what one might typically expect for elementary particles suggests that the inequality would happen with a curvature radius of about 10^{-8}cm, but this could clearly vary by a few orders of magnitude; thus my arguments work for radii of curvature from about 10^{-33} to 10^{-8}cm. Extrapolating, one would expect there to be uncertainty momentum relationships as well - assuming they give rise to similar expressions, then

$$G_{ab} = 8\pi T_{ab}(1 + (d/R)^2). \tag{2.13}$$

The position of the extra term in the field equations hides the fact that it is both massless and scalar; massless because, if it had mass, there would be an extra mass associated with any region one chose, so that if it had mass it would be infinite; scalar because it does not depend explicitly on any term involving indices only on $\max|R_{abcd}|$ and on an arbitrary volume. Another way of considering the masslessness of the new term is by realizing that it might appear to give rise to an infinite series:

$$G_{ab} = 8\pi T^1_{ab}(1 + (d/R)^2) = 8\pi T^2_{ab}(1 + (d/R)^2)^2 = 8\pi T^3_{ab}\ldots \tag{2.14}$$

this cannot occur as we are restricting ourselves to a given volume of radius d, if such a series did occur it would diverge leaving no field equations. The presence of the extra term has some unusual consequences such as space being more curved than just matter terms would lead one to believe. This is because of the uncertainty of local energy density arising from the curvature due to the matter fields. Without the new term, the field equations are observer dependent, because the observer is part of the matter fields, but with the new term there is an additional dependence as the observer sets up the volume of radius d in which he makes his measurements. It is interesting to note that space is curved more than one would expect classically, because of the quantum effects due to the presence of an observer and a volume in which he makes measurements.

The importance of this extra term can be seen by noting that it can be put in the form

$$T_{ab}(1 + (d/R)^2) = {}_{(m)}T_{ab} + {}_{(a)}T_{ab} = T^{total}_{ab}, \tag{2.15}$$

where ${}_{(a)}T_{ab}$ is a massless scalar field which has stress given by equation 2.4.

2.2 The QED Vacuum.

Quantum electrodynamics (QED) is the simplest quantum field theory which is physically realized. There are simpler QFT's which consist of just one or two scalar fields. QED is the quantum theory of the Maxwell field A_a and the Dirac spinor field ψ. There are four basic interactions in physics: electromagetic, weak, strong, and gravity; and QED is the quantum field theory which describes the first of these. The field theories corresponding to the other interactions get progressively harder to quantize and the methods by which their quantization is attempted usually follow successfull models in QED.

An example of a recent calculation of the vacuum properties of QED is that of Kong and Ravndal [202] (1998) who construct an effective field theory theory to calculate the properties of the QED vacuum at temperatures much less than the electron mass $T << m_e$. They do this by adding quantum fluctuation terms to the Maxwell lagrangian. Quantum fluctuations in the vacuum due to virtual electron loops can be included by extending the Maxwell lagrangian by additional non-renormalizable terms corresponding to the Uehling and Euler-Heisenberg interactions. By a redefinition of the electromagnetic field they show that the Uehling term does not contribute. The Stefan-Boltzmann energy density is thus found by them to be modified by a term proportional with T^8/m^4 in agreement with a semi-classical result of Barton [28] (1990), compare with the Toll-Scharnhorst effect §2.3 below. The speed of light in blackbody radiation is smaller than one. Similarly, the correction to the energy density of the vacuum between two metallic parallel plates diverges like $1/m^4 z^8$ at a distance from one of the plates $z \to 0$. While the integral of the regularized energy density is thus divergent, the regularized integral is finite and corresponds to a correction to the Casimir force which varies with the separation L between the plates as $1/m^4 L^8$. Kong and Ravndal [202] (1998) point out that this result is in seemingly disagreement with a previous result for the radiative correction to the Casimir force which gives a correction varying like $1/mL^5$ in a calculation using full QED. Marcic [241] (1990) discusses how vacuum energy alters photon electron scattering.

Xue and Sheng [411] (2001) study the fermionic vacuum energy of vacua with and without being applied an external magnetic field. The energetic difference of two vacua leads to the vacuum decaying and the vacuum-energy releasing. In the context of quantum field theories, they discuss why and how the vacuum energy can be released by spontaneous photon emissions and/or paramagnetically screening the external magnetic field. In addition, they quantitatively compute the vacuum energy released, the paramagnetic screening effect and the rate and spectrum of spontaneous photon emissions. They discuss the possibilities of experimentally detecting such a vacuum-energy releasing.

Nikolic [409] (2001) studies the possibility of electron-positron pair creation by an electric field by using only those methods in field theory the predictions of which are confirmed experimentally. These methods include the perturbative method of quantum electrodynamics and the bases of classical electrodynamics. Such an approach includes the back reaction. He finds that the vacuum is always stable, in the sense that pair creation, if occurs, cannot be interpreted as a decay of the vacuum, but rather as a decay of the source of the electric field or as a process similar to bremsstrahlung. He also finds that there is no pair creation in a static electric field, because it is inconsistent with energy conservation. He discusses the non-perturbative aspects arising from the Borel summation of a divergent perturbative expansion. He argues that the conventional methods that predict pair creation in a classical background electric field cannot serve even as approximations. He qualtatively discusses the analogy with the possibility of particle creation by a gravitational field.

2.3 The Toll-Scharnhorst Effect.

There is the problem of when any propagation can be considered to be light-like or null, I have previously discussed this in [287] (1987), [293] (1994) and [297] (1998). If one considers the propagation of light the speed c is its speed of propagation in a vacuum, as discussed in §1.3 above; there is never an absolute vacuum so that light never propagates at c. This might have significance for the spin states of light, for null propagation one of the spin states becomes a helicity state. There is also the problem of what happens to gravitational radiation co-moving with electromagnetic radiation. Even a photon has a gravitational field associated with itself, and at first sight one might expect it to co-move. When light moves from one medium to another, say air to water, its speed and direction change. So what happens to any co-moving gravitational radiation? It is unlikely to change by the same speed and direction as the photon as it interacts differently with matter, there might be some scattering processes that allows it to co-move, but this would be a coincidence.

A more concrete example of non-null light propagation is given by what is now called the Scharnhorst effect although similar studies go back to Toll [339] (1952). Beginning in the early 1950's, quantum field theoretic investigations have led to considerable insight into the relation between the vacuum structure and the propagation of light. Scharnhorst [312] (1998) notes that recent years have witnessed a significant growth of activity in this field of research. Loosely speaking the Scharnhorst effect is that boundaries change the nature of the quantum vacuum and this in turn changes the speed of propagation of light. Scharnhorst [311] (1990) considers QED in the presence of two parallel plates in a Casimir effect, see §3, type of configuration. The plates impose boundary conditions on photon vacuum fluctuations. In a physically reasonable approximation Scharnhorst calculates the two-loop corrections to the QED effective action. From this effective action Scharnhorst finds a change in the velocity of light propagating between and perpendicular to the plates. Barton [28] (1990) further discusses the effect which he says is due to the intensity of zero-point fields between parallel mirrors being less than in unbounded space.

Scharnhorst [312] (1998) notes that QED vacua under the influence of external conditions (background fields, finite temperature, boundary conditions) can be considered as dispersive media whose complex behaviour can no longer be described in terms of a single universal vacuum velocity of light c. After a short overview Scharnhorst [312] (1998) discusses *two* characteristic situations: *firstly* the propagation of light in a constant homogeneous magnetic field and *secondly* in a Casimir vacuum. The latter appears to be particularly interesting because the Casimir vacuum has been found to exhibit modes of the propagation of light with phase and group velocities larger than c in the low frequency domain $\omega << m$ where m is the electron mass. The impact of this result on the front velocity of light in a Casimir vacuum is discussed by means of the Kramers-Kronig relation.

Winterberg [359] (1998) notes that if the observed superluminal quantum correlations are disturbed by turbulent fluctuations of the zero-point vacuum energy field, with the perturbed energy spectrum assumed to obey the universal Kolmogoroff form derived above, which the correlations are conjectured to break. A directional dependence of this length would establish a preferred reference system at rest with zero-point energy. Assuming that the degree of turbulence is given by the small anisotropy of the cosmic microwave background radiation, a length similar to 60 Km. is derived above which the correlations would break.

Cougo-Pinto *et al* [89] (1999) consider the propagation of light in the QED vacuum between an unusual pair of parallel plates, namely: a perfectly conducting one ($\epsilon \to \infty$) and an infinitely permeable one ($\mu \to \infty$). For weak fields and in the soft photon approximation they show that the speed of light for propagation normal to the plates is smaller than its value in unbounded space in contrast to the original Scharnhorst [311] (1990) effect.

Liberati *et al* [225] (2000) consider the Scharnhorst effect at oblique incidence, calculating both photon speed and polarization states as functions of angle. The analysis is performed in the framework of nonlinear electrodynamics and they show that many features of the situation can be extracted solely on the basis of symmetry considerations. Although birefringence is common in nonlinear electrodynamics it is not universal; in particular they verify that the Casimir vacuum is not birefringent at any incidence angle. On the other hand, group velocity is typically not equal to phase velocity, though the distinction vanishes for special directions or if one is only working to second order in the fine structure constant. They obtain an "effective metric" that is subtly different from previous results. The disagreement is due to the way that "polarization sums" are implemented in the extant literature, and they demonstrate that a fully consistent polarization sum must be implemented via a bootstrap procedure using the effective metric one is attempting to define. Furthermore, in the case of birefringence, they show that the polarization sum technique is intrinsically an approximation.

These effects are too small by many orders of magnitude to be measured.

2.4 The Yang-Mills Vacuum.

Yang-Mills theory is a generalization of the Maxwell theory where instead of one vector field A_a there are several vector fields A_a^i interacting with each other subject to a group. Some studies of its quantum vacuum are listed below.

Casahorran and Ciria [68] (1995) discuss the stability of the vacuum in the presence of fermions.

Fumita [136] (1995) discusses the chiral, conformal and ghost number anomalies from the viewpoint of the quantum vacuum in Hamiltonian formalism. After introducing the energy cut-off, he derives known anomalies in a new way. He shows that the physical origin of the anomalies is the zero point fluctuation of bosonic or fermionic field. He first points out that the chiral $U(1)$ anomaly is understood as the creation of the chirality at the bottom of the regularized Dirac sea in classical electromagnetic field. In the study of the (1+1) dimensional quantum vacuum of matter field coupled to the gravity, he gives a physically intuitive picture of the conformal anomaly. The central charges are evaluated from the vacuum energy. He clarifies that the non-Hermitian regularization factor of the vacuum energy is responsible for the ghost number anomaly.

Using a variational method Cea [70] (1996) evaluates the vacuum energy density in the one-loop approximation for three dimensional abelian and non-abelian gauge theories interacting with Dirac fermions. It turns out that the states with a constant magnetic condensate lie below the perturbative ground state only in the case of three dimensional quantum electrodynamics with massive fermions.

Ksenzov [209] (1997) shows that some non-perturbative properties of the vacuum are described by the quantum fluctuations around the classical background with zero canonical momentum. The vacuum state that he built was in in the framework of the σ-models in two dimensions.

Natale and da Silva [256] (1997) show that if a gauge theory with dynamical symmetry breaking has non-trivial fixed points, they will correspond to extrema of the vacuum energy. This relationship provides a different method to determine fixed points.

Tiktopoulos [337] (1997) applies variational (Rayleigh-Ritz) methods to local quantum field theory. For scalar theories the wave functional is parametrized in the form of a superposition of Gaussians and the expectation value of the Hamiltonian is expressed in a form that can be minimized numerically. A scheme of successive refinements of the superposition is proposed that might converge to the exact functional. As an illustration, he works out a simple numerical approximation for the effective potential is worked out based on minimization with respect to five variational parameters. A variational principle is formulated for the fermion vacuum energy as a functional of the scalar fields to which the fermions are coupled. The discussion in this paper is given for scalar and fermion interactions in 1+1 dimensions. The extension to higher dimensions encounters a more involved structure of ultraviolet divergences and he defers it to future work.

Kosyakov [408] (1998) discusses in a systematical way exact retarded solutions to the classical SU(N) Yang-Mills equations with the source composed of several colored point particles. He reviews anew method of finding such solutions. Relying on features of the solutions, he suggests a toy model of quark binding. According to this model, quarks forming a hadron are influenced by no confining force in spite of the presence of a linearly rising term of the potential. The large-N dynamics of quarks conforms well with Witten's phenomenology. On the semiclassical level, hadrons are color neutral in the Gauss law sense. Nevertheless, a specific multiplet structure is observable in the form of the Regge sequences related to infinite-dimensional unitary representations of $SL(4, R)$ which is shown to be the color gauge group of the background field generated by any hadron. The simultaneous consideration of $SU(N)$, $SO(N)$, and $Sp(N)$ as gauge groups offers a plausible explanation of the fact that clusters containing two or three quarks are more stable than multiquark clusters.

Gogohia and Kluge [148] (2000) using the effective potential approach for composite operators, they formulate a general method of calculation of the non-perturbative Yang-Mills vacuum energy density in the covariant gauge QCD ground state quantum models. The Yang-Mills vacuum energy density is defined as an integration of the truly nonperturbative part of the full gluon propagator over the deep infrared region (soft momentum region). A nontrivial minimization procedure makes it possible to determine the value of the soft cutoff in terms of the corresponding nonperturbative scale parameter, which is inevitably present in any nonperturbative model for the full gluon propagator. They show for specific models of the full gluon propagator that explicitly use of the infrared enhanced and finite gluon propagators lead to the vacuum energy density which is finite, always negative and it has no imaginary part (stable vacuum), while the infrared vanishing propagators lead to unstable vacuum and therefore they are physically unacceptable.

Guendelman and Portnoy [158] (2000) review and analyze the stability of the vacuum of several models. 1) In the standard Glashow-Weinberg-Salam (GWS) model they review the instability towards the formation of a bubble of lower energy density and how the rate of such bubble formation process compares with the age of the Universe for the known values of the GWS model. 2)They also review the recent work by Guendelman concerning the vacuum instability question in the context of a model that solves the cosmological constant problem. Gundelman and Portnoy [158] claim that it turns out that in such model the same physics that solves the cosmological constant problem makes the vacuum stable. 3)They review their recent work concerning the instability of elementary particle embedded in our vacuum, towards the formation of an infinite Universe. they say that such process is not catastrophic and it leads to a "bifurcation type" instability in which our Universe is not eaten by a bubble (instead a baby universe is born). This universe does not replace our Universe rather it disconnects from it (via a wormhole) after formation.

Polychronakos [277] (2000) points out that the space noncommutativity parameters $\theta^{\mu\nu}$ in noncommutative gauge theory can be considered as a set of superselection parameters, in analogy with the θ-angle in ordinary gauge theories. As such, they do not need to enter explicitly into the action. He suggests a simple generic formula to reproduce the Chern-Simons action in noncommutative gauge theory, which reduces to the standard action in the commutative limit but in general implies a cascade of lower-dimensional Chern-Simons terms. The presence of these terms in general alters the vacuum structure of the theory and nonstandard gauge theories can emerge around the new vacua.

2.5 The QCD Vacuum.

Quantum chromdynamics (QCD) is a particular Yang-Mills theory in which the group structure is taken to be of a specific form which can describe the strong interactions of particle physics. The coupling constants in the theory are large, which means that, unlike QED, perturbations in these coupling constants do not converge. Thus the question arises as to what the nonperturbative vacuum could be, as discussed in Goghia and Kluge [148] (2000) §2.4 above and the papers discussed below. Grundberg and Hanson [154] (1994) use arguments taken from the electrodynamics of media to deduce the QCD trace anomaly from the expression for vacuum energy in the presence of an external colour magnetic field.

One way of approaching nonperturbative vacuum energy is via instanton effects, see §2.12 below and Shuryak and Schäfer [317] (1997) who review recent progress in understanding the importance of instanton effects in QCD. Instantons explain the appearance of a non-perturbative vacuum energy density, as calculated from correlation functions as a bridge between vacuum and hadronic structures.

Goghia *et al* [149] (1998) use the effective potential approach for composite operators to formulate the quantum model of the QCD vacuum. It is based on the existence and importance of the nonperturbative q^{-4}-type dynamical, topologically nontrivial excitations of the gluon field configuration. The QCD vacuum is found to be stable since the vacuum energy density has no imaginary part. Moreover, they discover a possible stationary ground state of the nonperturbative Yang-Mills (quenched QCD) vacuum. The vacuum energy density at stationary state depends on a scale at which nonperturbative effects become important. The quark part of the vacuum energy density depends in addition on the constant of integration of the corresponding Schwinger-Dyson equation. The value of the above mentioned scale is determined from the bounds for the pion decay constant in the chiral limit. Their value for the chiral QCD vacuum energy density is one order of magnitude bigger than the instanton based models can provide while a fair agreement with recent phenomenological and lattice results for the chiral condensate is obtained.

Kosyakov [408] (1998) approaches QCD through exact solutions of the Yang-Mills-Wong equations, see the previous section §2.4.

Schmidt and Yang [307] (1999) discuss QCD condensate contributions to the gluon propagator both in the fixed-point gauge and in covariant gauges for the external QCD vacuum gluon fields with the conclusion that a covariant gauge is essential to obtain a gauge invariant QCD vacuum energy density difference and to retain the unitarity of the quark scattering amplitude. The gauge-invariant QCD condensate contributions to the effective one-gluon exchange potential are evaluated by using the effective gluon propagator which produces a gauge-independent quark scattering amplitude.

Luo [232] (1998) investigates the vacuum properties of lattice QCD with staggered quarks by an efficient simulation method. He presents data for the quark condensate with flavour number $N_f = 0,\ 1,\ 2,\ 3,\ 4$ and many quark masses, including the vacuum energy in the chiral limit. Obvious sea quark effects are observed in some parameter space. He also describes a mechanism to understand this and a formula relating the chiral condensate and zero modes.

Velkovsky and Shuryak [345] (1998) calculate the contribution of the instanton – anti-instanton ($I\bar{I}$) pairs to the vacuum energy of QCD-like theories with N_f light fermions using the saddle point method. They find a qualitative change of the behavior: for $N_f \geq 6$ it starts to oscillate with N_f. Similar behaviour was known for quantum mechanical systems interacting with fermions. They discuss the possible consequences of this phenomenon, and its relation to the mechanism of chiral symmetry breaking in these theories. They also discuss the asymptotics of the perturbative series associated with the $I\bar{I}$ contribution, comparing their results with those in literature.

Ksenzov [208] (2000) describes the non-perturbative part of the vacuum energy density for static configuration in pure SU(2) Yang-Mills theory, and also he constructs a vacuum state.

Montero *et al* [250] (1997) claim to find that nonperturbative infrared finite solutions for the gluon polarization tensor. The possibility that gluons might have a dynamically generated mass is supported by recent Monte Carlo simulation on the lattice. These solutions differ among themselves, due to different approximations performed when solving the Schwinger-Dyson equations for the gluon polarization tensor. Only approximations that minimize energy are meaningful, and, according to this, they compute an effective potential for composite operators as a function of these solutions in order to distinguish which one is selected by the vacuum.

Paniak *et al* [266] (1996) address the issue of topological angles in the context of two dimensional SU(N) Yang-Mills theory coupled to massive fermions in the adjoint representation. Classification of the resulting multiplicity of vacua is carried out in terms of asymptotic fundamental Wilson loops, or equivalently, charges at the boundary of the world. They explicitly demonstrate that the multiplicity of vacuum states is equal to N for SU(N) gauge group. Different worlds of the theory are classified by the integer number k=0,1,...N-1 (superselection rules) which plays an analogous role to the θ parameter in QCD. Via two completely independent approaches they study the physical properties of these unconnected worlds as a function of k. *Firstly* they apply the well known machinery of the loop calculus in order to calculate the effective string tensions in the theory as function of k. The *second* way of doing the same physics is the standard particle/field theoretic calculation for the binding potential of a pair of infinitely massive fermions. They also calculate the vacuum energy as function of k.

An equivariant BRST-construction is used by Schaden [306] (1998) to define the continuum SU(3) gauge theory on a finite torus. Schaden corroborate previous results using renormalization group techniques by explicitly computing the measure on the moduli-space of the model with 3 quark flavours to two loops. Schaden finds that the correction to the maximum of the one-loop effective action is indeed of order g^2 in the critical covariant gauge. The leading logarithmic corrections from higher loops are also shown to be suppressed by at least one order of g^2. Schaden therefore is able to relate the expectation value of the moduli to the asymptotic scale parameter of the modified minimal subtraction scheme. An immediate consequence is the determination of the non-perturbative proportionality constant in the relation between the vacuum expectation value of the trace of the energy momentum tensor and Λ_{QCD} for the modified minimal subtraction scheme with three quark flavours. The result compares favorably with phenomenological estimates of the gluon condensate from QCD sum rules for the charmonium system and Λ_{QCD} from τ-decay.

Gabadadze and Shifman [138] (2000) show that large N gluodynamics to have a set of metastable vacua with the gluonic domain walls interpolating between them. The walls may separate the genuine vacuum from an excited one, or two excited vacua which are unstable at finite N (here N is the number of colours). One might attempt to stabilize them by switching on the axion field. They study how the light quarks and the axion affect the structure of the domain walls. In pure gluodynamics (with the axion field) the axion walls acquire a very hard gluonic core. Thus, they deal with a wall "sandwich" which is stable at finite N. In the case of the minimal axion, the wall "sandwich" is in fact a "$2-\pi$" wall, i.e., the corresponding field configuration interpolates between identical hadronic vacua. The same properties hold in QCD with three light quarks and very large number of colours. However, in the realistic case of three-colour QCD the phase corresponding to the axion field profile in the axion wall is screened by a dynamical phase associated with the η', so that the gluon component of the wall is not excited. They propose a toy Lagrangian which models these properties and allows one to get exact solutions for the domain walls.

2.6 The SUSY Vacuum.

From the point of view of inquiring what the quantum vacuum is, supersymmetry is important. In supersymmetric theories the overall infinite vacuum energies of bosons and fermions cancel out term by term. Of course if supersymmetry is realized in nature then it is badly broken, whether the above type of term by term cancellation occurs in broken supersymmetry is an open question, as the correct way to break supersymmetry to establish contact with the physical world has not been found. A description of the term by term cancellations is given in Peskin and Schroeder [273] page 796 (1995) who say:

> We have noted already that bosonic fields give positive contributions to the vacuum energy through their zero-point energy, and fermionic fields give negative contributions. We now see that, in a supersymmetric model, these contributions cancel exactly, not only at the leading order but to all orders in perturbation theory.

Following Carroll [67] (2000) SUSY is a spacetime symmetry relating fermions and bosons to each other. Just as ordinary symmetries are associated with conserved charges, supersymmetry is associated with "supercharges" Q_α, where α is a spinor index. As with ordinary symmetries, a theory might be supersymmetric even though a given state is not supersymmetric; a state which is annihileted by the supercharges, $Q_\alpha|\psi>=0$, presevres supersymmetry, while states with $Q-\alpha|\psi>\neq 0$ are said to "spontaneously break" SUSY.

Consider "globally supersymmetric' theories, which are defined in flat spacetime. Unlike most nongravitating field theories, in supersymmetry the total energy of a state has an absolute meaning: the Hamiltonian is related to the supercharges in a straightforward way:

$$\mathcal{H}=\sum_\alpha\{Q_\alpha, Q_\alpha^\dagger\}, \tag{2.16}$$

where brackets represent the anticommutator. Thus, in a completely supersymmetric state (in which $Q - \alpha|\psi >= 0 \forall \alpha$), the energy vanishes automatically, $< \psi|H|\psi) = 0$. More concretely, in a given supersymmetric theory one can explicitly calculate the contributions to the vacuum energy from vacuum fluctuations and the scalar potential V. In the case of vacuum fluctuations, contributions from bosons are exactly canceled by equal and opposite contributions when supersymmetry is unbroken. Meanwhile, the scalar-field potential in supersymmetric theories takes on a special form; scalar fields ψ^i must be complex (to match the degrees of freedom of the fermions), and the potential is derived from a function called the superpotential $W(\psi^i)$ which is necessarily holomorphic (written in terms of ψ^i and not its complex conjugate $\bar{\psi}^i$). In the simple Wess-Zumino models of spin-0 and spin-1/2 fields, for example, the scalar potential is given by

$$V(\psi^i, \bar{\psi}^j) = \sum_1 |\partial_i W|^2, \tag{2.17}$$

where $\partial_i W = \partial W / \partial \psi^i$. In such a theory, one can show that SUSY will be unbroken only for values of ψ^i such that $\partial_i W = 0$, implying that $V(\psi^i, \bar{\psi}^j) = 0$.

So the vacuum energy of a supersymmetric state in a globally supersymmetric theory will vanish. This represents rather less progress than it might appear at first sight, since *firstly* supersymmetric states manifest a degeneracy in the mass spectrum of bosons and fermions, a feature not apparent in the observed world; and *secondly* the above results imply that non-supersymmetric states have a positive definite vacuum energy. Indeed, in a state where SUSY was broken at an energy scale M_{SUSY} it would be expected that the corresponding vacuum energy $\rho_\Lambda \sim M_{SUSY}^4$. In the real world, the fact that the accelerator experiments have not discovered superpartners for the known particles of the Standard Model implies that M_{SUSY} is of order 10^3 GeV or higher. Thus a discrepancy

$$\frac{M_{SUSYU}}{M_{VAC}} \geq 10^{15} \tag{2.18}$$

is left. Comparison of this discrepancy with the naive discrepancy of Carroll [67] eq.54 (2000), is the source of the claim that SUSY can solve the cosmological constant problem halfway, at least on a log scale.

This analysis is strictly valid only in flat space. In curved spacetime, the global transformations of ordinary supergravity are promoted to the position dependent, or gauge, transformations of supergravity. In this context the Hamiltonian and the supersymmetry generators play different roles than that in flat spacetime, but it is still possible to express the vacuum energy, here meaning cosmological constant, in terms of the scalar field potential $V(\phi^i, \bar{\phi}^j)$. In supergravity V depends not only on the superpotentioal $W(\phi^i)$, but also on a "Kähler potential" $K(\phi^i, \bar{\phi}^j)$, and the Kähler metric $K_{i\bar{j}}$ constructed from the Kälher potential by $K_{i\bar{j}} = \partial^2 K / \partial \phi^i \partial \bar{\phi}^j$. (The basic role of the Kälher metric is to define the kinetic term for the scalars, which takes the form $g^{\mu\nu} K_{i\bar{j}} \partial_\mu \phi^i \partial_\nu \bar{\phi}^j$.)

The scalar potential is

$$V(\phi^i, \bar{\phi}^j) = \exp\left(\frac{K}{M_{Pl}^2}\right)\left[K^{i\bar{j}}(D_iW)(D_{\bar{j}}\bar{W}) - 3M_{PL}^{-2}|W|^2\right], \tag{2.19}$$

where D_iW is the Kähler derivative,

$$D_iW = \partial_i W + M_{Pl}^{-2}(\partial_i K)W. \tag{2.20}$$

Note that, if we take the canonical Kähler metric $K_{i\bar{j}} = \delta_{i\bar{j}}$, in the limit $M_{Pl} \to \infty (G \to 0)$ the first term in square brackets reduces to the flat result equation 2.17. But with gravity, in addition to the non-negative first term there is a second term providing a non-positive contribution. Supersymmetry is unbroken when $D - IW = 0$; the effective cosmological constant is thus non-positive. One is, in equation 2.19 therefore free to imagine a scenario in which supersymmetry is broken in exactly the right way, such that the two terms in parentheses cancel to fantastic accuracy, but only at the cost of an unexplained fine-tuning. At the same time, supergravity is not by itself a renormalizable quantum theory, and therefore it might not be reasonable to hope that a solution can be found purely within this context. Some recent work on SUSY vacua includes the below.

Brignole *et al* [55] (1995) construct $N = 1$ supergravity models where the gauge symmetry and supersymmetry are both spontaneously broken, with naturally vanishing classical vacuum energy, here again meaning the cosmological constant, and unsuppressed Goldstino components along gauge non-singlet directions. They discuss some physically interesting situations where such a mechanism could play a role, and identify the breaking of a grand-unified gauge group as the most likely possibility. They show that, even when the gravitino mass is much smaller than the scale m_X of gauge symmetry breaking, important features can be missed if they first naively integrate out the degrees of freedom of mass $\mathcal{O}(m_X)$, in the limit of unbroken supersymmetry, and then describe the super-Higgs effect in the resulting effective theory. They also comment on possible connections with extended supergravities and realistic four-dimensional string constructions.

Anselm and Berezhiani [16] (1996) produce an analogy to the case of axion, which converts the θ-angle into a dynamical degree of freedom, they try to imagine a situation where the quark mixing angles turn out to be dynamical degrees of freedom (pseudo-Goldstone bosons), and their vacuum expectation values are obtained from the minimization of the vacuum energy. They present an explicit supersymmetric model with horizontal symmetry, where such a mechanism can be realized. It implies one relation between the quark masses and the CKM mixing angles: $s_{13}s_{23}/s_{12} = (m_s/m_b)^2$, which is fulfilled within present experimental accuracy. They believe, however, that the idea might be more general than this concrete model, and it can be implemented in more profound frameworks.

Lythe and Stewart [236] (1996) claim that supersymmetric theories can develop a vacuum expectation value of $M >> 10^3$ GeV, when the temperature of the early universe falls below some number related to this there is thermal inflation, see also §4.2.

Das and Pernice [91] (1997) propose a new mechanism for symmetry breaking which naturally avoids the constraints following from the usual theorems of symmetry breaking. In the context of supersymmetry, for example, the breaking might be consistent with a vanishing vacuum energy. A 2+1 dimensional super-symmetric gauge field theory is explicitly shown to break super-symmetry through this mechanism while maintaining a zero vacuum energy. They claim that this mechanism might provide a solution to two long standing problems *firstly* dynamical super-symmetry breaking and *secondly* the cosmological constant problem.

Matsuda [243] (1996) examines the phase structures of the supersymmetric $O(N)$ σ-model in two and three dimensions by using the tadpole method. Using this simple method, the tadpole calculation is largely simplified and the characteristics of this theory become clear. He also examines the problem of the fictitious negative energy state.

Oda [261] (1997) studies the vacuum structures of supersymmetric (SUSY) Yang-Mills theories in $1+1$ dimensions with the spatial direction compactified. SUSY allows only periodic boundary conditions for both fermions and bosons. By using the Born-Oppenheimer approximation for the weak coupling limit, Oda finds that the vacuum energy vanishes, and hence the SUSY is unbroken. He studies other boundary conditions, especially the antiperiodic boundary condition for fermions which is related to the system in finite temperatures. In that case Oda finds for gaugino bilinears a nonvanishing vacuum condensation which indicates instanton contributions.

Chernyak (1999) [76] shows that there is no chirally symmetric vacuum state in the $\mathcal{N}=1$ supersymmetric Yang-Mills theory. The values of the gluino condensate and the vacuum energy density are found out through a direct instanton calculation. A qualitative picture of domain wall properties is presented, and a new explanation of the phenomenon of strings ending on the wall is proposed.

Riotto [282] (1998) says that the scale at which supersymmetry is broken and the mechanism by which supersymmetry breaking is fed down to the observable sector has rich implications on the way nature might have chosen to accomplish inflation, see also §4.4. Riotto discusses a simple model for slow rollover inflation which is minimal in the sense that the inflaton might be identified with the field responsible for the generation of the μ-term. Inflation takes place at very late times and is characterized by a very low reheating temperature. This property is crucial to solve the gravitino problem and might help to ameliorate the cosmological moduli problem. The COBE normalized value of the vacuum energy driving inflation is naturally of the order of 10^{11} GeV. This favours the N=1 supergravity scenario where supersymmetry breaking is mediated by gravitational interactions. Nonetheless, smaller values of the vacuum energy are not excluded by present data on the temperature anisotropy and the inflationary scenario may be implemented in the context of new recent ideas about gauge mediation where the standard model gauge interactions can serve as the messengers of supersymmetry breaking. In this class of models supersymmetry breaking masses are usually proportional to the F-term of a gauge singlet superfield. The same F-term might provide the vacuum energy density necessary to drive inflation. The spectrum of density perturbations is characterized by a spectral index which is significantly displaced from one. The measurements of the temperature anisotropies in the cosmic microwave background radiation at the accuracy expected to result from the planned missions will be able to confirm or disprove this prediction and to help in getting some deeper insight into the nature of supersymmetry breaking.

2.7 The Supergravity Vacuum.

Supergravity was first done in four dimensions with extra asymmetric fields. Then it was noted that it could be more conveniently expressed as an eleven dimensional theory, in which the extra seven dimensions were simply a notational device which simplified its presentation. Freund and Rubin [133] (1980), see also Roberts [291] (1991), took the extra dimensions to be there and assumed a non-zero asymmetric field F, this can be thought of as assuming a non-zero vacuum. The non-zero field causes the four external and seven internal dimensions to "curl up" into maximally symmetric spaces with the constant of curvature related to the asymmetric field F. For the four external dimensions spacetime is anti-deSitter spacetime with the constant of curvature being the cosmological constant with a value outside that allowed for by observations. Recent work on the Freund-Rubin ansatz includes Lütken and Ordonez [233, 234] (1987) who say that for the Freund-Rubin solution of $d = 11$ supergravity to have small extra dimensions it is necessary for the corresponding spacetime to be anti-deSitter spacetime with an unphysically large cosmological constant. The aim of their two papers is to investigate whether there are solutions which are a product of Minkowski spacetime and have small extra dimensions, when quantum mechanical vacuum energy is taken into account. The authors' conclusion is that, for the case of the vanishing Freund-Rubin parameter, there are none. In their *first* paper the effective potential for gravitinos, in $d = 11$ supergravity on the background of a product of Minkowski spacetime and the seven-sphere, is calculated by a background field method to one loop. The effective potential is lower than that for lower spin fields, but smaller than that for gravitons. In the *second* paper the effective potential for the bosonic sector is calculated, for the case of vanishing Freund-Rubin parameter; the case of nonvanishing Freund-Rubin parameter is intractable by this method. It is found that the three-index antisymmetric gauge field F has a lower effective potential than the gravitino, and that the graviton has a higher effective potential than that of the gravitino. Fré *et al* [130] (1996) exhibit generic partial supersymmetry breaking of $N = 2$ supergravity with zero vacuum energy and with surviving unbroken arbitrary gauge groups, and they give specific examples.

Ellwanger [116] (1995) constructs supergravity models are constructed in which the effective low energy theory contains only "super-soft" explicit supersymmetry breaking: masses of the scalars and pseudoscalars within a multiplet are split in opposite directions. With this form of supersymmetry breaking the radiative corrections of the matter sector to the vacuum energy are bounded by $\mathcal{O}(M^4_{Susy})$ to all orders in perturbation theory, and he requires $StrM^2 = 0$

including the hidden sector. The models are based on Kähler potentials obtained in recent orbifold compactifications, and he describes the construction of realistic theories.

2.8 String and Brane Vacua.

The string vacuum is similar to the supergravity vacuum §2.7 above, in that the nature of the vacuum is related to the splitting of space into four spacetime dimensions and internal dimensions. For example Candelas *et al* [64] (1985) note that the vacuum state should be of the form $M_4 \times K$, where M_4 is the spactime manifold and K is some six dimensional compact manifold. To determine K they say one needs geometry of the above form with M_4 maximally symmetric. They also say one needs unbroken $N = 1$ supersymmetry in $d = 4$ as this might resolve the gauge hierarchy and Dirac large number problem. Furthermore the gauge group and fermionic spectrum should be realistic. The manifold forced on Candelas *et al* by phenomenological considerations in the field theory limit are precisely those manifolds on which it is possible to formulate a consistent string theory. They say that indications are that the candidate vacuum configurations singled out by phenomenological considerations obey the equations of motion.

Largely following Carroll [67] §4.2 (2000) note that unlike supergravity, string theory appears to be a consistent and well-defined theory of quantum gravity, and therefore calculating the value of the cosmological constant should at least in principle be possible. Cosmological constant here means something related to potentials V, and thus is subject to the caveats at the end of §2.1¶2. On the other hand, the number odd vacuum states seems to be quite large, and none of them (to the best of our current knowledge) features three large spatial dimensions, broken supersymmetry, and a small cosmological constant. At the same time, there are reasons to believe that any realistic vacuum of string theory must be strongly coupled (although it is not clear what to, perhaps self-coupling is meant); therefore, our inability to find an appropriate solution might simply be due to the technical difficulty of the problem.

String theory is naturally formulated in more than four spacetime dimensions. Studies of duality symmetries have revealed that what used to be thought of as five distinct ten-dimensional superstring theories - Type I, Types IIA and IIB, and heterotic theories based on gauge groups $E(8) \times E(8)$ and $SO(32)$ - are along with eleven-dimensional supergravity, different low-energy weak-coupling limited of a single underlying theory, sometimes known as M-theory. In each of these six cases, the solution with the maximum number of uncompactified flat flat spacetime dimensions is a stable vacuum preserving all of the supersymmetry. To bring the theory closer to the world that is observed, the extra dimensions can be compactified on a manifold whose Ricci tensor vanishes. there are a large number of possible compactifications, many who preserve some but not all of the original supersymmetry. If enough SUSY is preserved, the vacuum energy will remain zero; generically there will be a manifold of such states, known as the moduli space.

Of course, to describe our world we want to break all supersymmetry in order for there to be correspondence with the observed world. Investigations in contexts where this can be done in a controlled way have found that the induced cosmological constant vanishes at the classical level, but a substantial vacuum energy is typically induced by quantum corrections. Moore [251] (1987) has suggested that Atkin-Lehner symmetry, which relates strong and weak coupling on the string world sheet, can enforce the vanishing of the one-loop quantum contribution in certain models; generically however there would still be an appreciable contribution at two loops.

Thus, the search is still on for a four-dimensional string theory vacuum with broken supersymmetry and vanishing or very small cosmological constant, see Dine [100] (1999) below for a general discussion of the vacuum problem in string theory. Carroll [67] (2000) notes that the difficulty of achieving this in conventional models has inspired a number of more speculative models:

1. In three spacetime dimensions supersymmetry can remain unbroken, maintaining a zero cosmological constant, in such a way as to break the mass degeneracy between bosons and fermions. This mechanism relies crucially on special properties of spacetime in (2+1) dimensions, but in string theory it sometimes happens that the strong-coupling limit of one theory is another theory in a higher dimension.

2. More generally, it is now understood that (at least in some circumstances) string theory obeys the "holographic principle", the idea that a theory with gravity in D dimensions is equivalent to a theory without gravity in $D-1$ dimensions. In a holographic theory, the number of degrees of of freedom in a region grows as the area of its boundary, rather than its volume. Therefore, the conventional computation of the cosmological constant due to vacuum fluctuations conceivably involves a vast overcounting of degrees of freedom. It might be imagined that a more correct counting would yield a much smaller estimate of the vacuum energy, although no reliable calculation has been done yet. See also Thomas [336] (2000).

3. The absence of manifest SUSY in our world leads us to ask whether the beneficial aspect of canceling contributions to the vacuum energy could be achieved even without a truly supersymmetric theory. Kachru *et al* (1998), see also this section below, have constructed such a string theory, and argue that the perturbative contributions to the cosmological constant should vanish (although the actual calculations are somewhat delicate, and not everyone agrees.) If such a model could be made to work, it is possible that the small non-perturbative effects could generate a cosmological constant of an astrophysically plausible magnitude.

4. A novel approach to compactification starts by imagining that the fields of the standard model are confined to a (3+1)-dimensional manifold (or "brane", in string theory parlance) embedded in a larger space. While gravity is harder to confine to a brane, phenomenologically acceptable scenarios can be constructed if either the extra dimensions are any size

less than a millimeter, or if there is significant spacetime curvature in a non-compact extra dimension. Although these scenarios do not offer a simple solution to the cosmological constant problem, the relationship between the vacuum energy and the expansion rate can differ from our conventional expectation, and one is free to imagine that further study might lead to a solution in this context.

Buchbinder *et al* [374] (1992) discuss some techniques used in string theory calculations.

Chang and Dowker [73] (1992) calculate vacuum energy for a free, conformally-coupled scalar field on the orbifold space-time $\mathrm{R}\times S2/\Gamma$ where Γ is a finite subgroup of O(3) acting with fixed points. The energy vanishes when Γ is composed of pure rotations but not otherwise. It is shown on general grounds that the same conclusion holds for all even-dimensional factored spheres and the vacuum energies are given as generalized Bernoulli functions (i.e. Todd polynomials). The relevant ζ- functions are analyzed in some detail and several identities are incidentally derived. Chang and Dowker give a general discussion in terms of finite reflection groups.

Cornwall and Yan [88] (1995) give a quantitative (if model-dependent) estimate of the relation between the string tension and a gauge-invariant measure of the Chern-Simons susceptibility, due to vortex linkages, in the absence of a Chern-Simons term in the action, based on a model of the d=3 SU(2) pure gauge theory vacuum as a monopole-vortex condensate, they give a quantitative (if model-dependent) estimate of the relation between the string tension and a gauge-invariant measure of the Chern-Simons susceptibility, due to vortex linkages, in the absence of a Chern-Simons term in the action. They also give relations among these quantities and the vacuum energy and gauge-boson mass. Both the susceptibility and the string tension come from the same physics: namely the topology of linking, twisting, and writhing of closed vortex strings. The closed-vortex string is described via a complex scalar field theory whose action has a precisely-specified functional form, inferred from previous work giving the exact form of a gauge theory effective potential at low energy.

Lopez and Nanopoulos [230] (1995) show that the presence of an anomalous $\mathrm{U_A}(1)$ factor in the gauge group of string derived models might have the new and important phenomenological consequence of allowing the vanishing of $\mathrm{Str}\,\mathcal{M}^2$ in the "shifted" vacuum, that results in the process of canceling the anomalous $\mathrm{U_A}(1)$. The feasibility of this effect seems to be enhanced by a vanishing vacuum energy, and by a "small" value of $\mathrm{Str}\,\mathcal{M}^2$ in the original vacuum. In the class of free-fermionic models with vanishing vacuum energy that they focus on, a necessary condition for this mechanism to be effective is that $\mathrm{Str}\,\mathcal{M}^2 > 0$ in the original vacuum. They say that a vanishing $\mathrm{Str}\,\mathcal{M}^2$ ameliorates the cosmological constant problem and is a necessary element in the stability of the no-scale mechanism, again they take the cosmological constant is taken to be related to vacuum energy.

Naculich [255] (1995) notes that Z-strings in the Weinberg-Salam model including fermions are unstable for all values of the parameters. The cause of this instability is the fermion vacuum energy in the Z-string background. Z-strings with non-zero fermion densities, however, might still be stable.

Leontaris and Tracas [219] (1996) analyse the constraints from duality invariance on effective supergravity models with an intermediate gauge symmetry. Requiring vanishing vacuum energy and invariance of the superpotential couplings, they find that modular weights of the matter and Higgs fields are subject to various constraints. In addition, the vacuum expectation values of the Higgs fields breaking the intermediate gauge group, are determined in terms of their modular weights and the moduli. They also examine the possibility of breaking the intermediate gauge symmetry radiatively.

Berntsssen *et al* [369] (1997) study the Casimir effect for a string.

Kim and Rey [197] (1997) study matrix theory on an orbifold and classical two-branes therein with particular emphasis to heterotic M(atrix) theory on S_1/Z_2 relevant to strongly coupled heterotic and dual Type IA string theories. By analyzing orbifold condition on Chan-Paton factors, they show that three choice of gauge group are possible for heterotic M(atrix) theory: SO(2N), SO(2N+1) or USp(2N). By examining area preserving diffeomorphism that underlies the M(atrix) theory, they find that each choices of gauge group restricts possible topologies of two-branes. The result suggests that only the choice of SO(2N) or SO(2N+1) groups allows open two-branes, hence, relevant to heterotic M(atrix) theory. They show that requirement of both local vacuum energy cancellation and of worldsheet anomaly cancellation of resulting heterotic string identifies supersymmetric twisted sector spectra with sixteen fundamental representation spinors from each of the two fixed points. Twisted open and closed two-brane configurations are obtained in the large N limit.

Aldazabal *et al* [10] (1998) consider $D = 6, N = 1, Z_M$ orbifold compactifications of heterotic strings in which the usual modular invariance constraints are violated. They argue that in the presence of non-perturbative effects many of these vacua are nevertheless consistent. The perturbative massless sector can be computed explicitly from the perturbative mass formula subject to an extra shift in the vacuum energy. They say that this shift is associated to a non-trivial antisymmetric B-field flux at the orbifold fixed points. The non-perturbative piece is given by five-branes either moving in the bulk or stuck at the fixed points, giving rise to Coulomb phases with tensor multiplets. The heterotic duals of some Type IIB orientifolds belong to this class of orbifold models. Aldazabal *et al* also discuss how to carry out this type of construction to the $D = 4, N = 1$ case and specific $Z_M \times Z_M$ examples are presented in which non-perturbative transitions changing the number of chiral generations do occur.

Buonanno *et al* [60] (1998) discusses how string vacuum energy might give rise to prebig bang bubbles.

Kachru *et al* [189] (1998) present a nonsupersymmetric orbifold of type II string theory and show that it has vanishing cosmological constant at the one and two loop level, here the cosmological constant is taken to be related to the vacuum energy. They argue heuristically that the cancellation persists at higher loops.

Kachru and Silverstein [188] (1998) propose and test correspondences between 4D QFT's with N=2,1,0 conformal invariance and type IIB string theory on various orbifolds of $AdS_5 \times S^5$. This allows them to translate the problem of finding stable nontrivial nonsuper string background into the problem of realizing nontrivial renormalization group fixed point QFT's on branes. Renormalization group fixed lines in this context correspond to string theories in which no vacuum energy is generated quantum mechanically.

Kawamura [194] (1998) studies the magnitudes of soft masses in heterotic string models with anomalous U(1) symmetry model-independently. In most cases, D-term contribution to soft scalar masses is expected to be comparable to or dominant over other contributions provided that supersymmetry breaking is mediated by the gravitational interaction and/or an anomalous U(1) symmetry and the magnitude of vacuum energy is not more than of order $m_{3/2}^2 M^2$.

Angelantonj *et al* [15] (1999) study open descendants of non-supersymmetric type IIB asymmetric (freely acting) orbifolds with zero cosmological constant, here again the cosmological constant is taken to be related to the vacuum energy. A generic feature of these models is that supersymmetry remains unbroken on the brane at all mass levels, while it is broken in the bulk in a way that preserves Fermi-Bose degeneracy in both the massless and massive (closed string) spectrum. This property remains valid in the heterotic dual of the type II model but only for the massless excitations. A possible application of these constructions concerns scenarios of low-energy supersymmetry breaking with large dimensions.

Bianchi *et al* [33] (1999) determine the spectrum of D-string bound states in various classes of generalized type I vacuum configurations with sixteen and eight supercharges. They say that the precise matching of the BPS spectra confirms the duality between unconventional type IIB orientifolds with quantized NS-NS antisymmetric tensor and heterotic CHL models in D=8. A similar analysis puts the duality between type II (4,0) models and type I strings *without open strings* on a firmer ground. The analysis can be extended to type II (2,0) asymmetric orbifolds and their type I duals that correspond to unconventional K3 compactifications. Finally they discuss BPS-saturated threshold corrections to the corresponding low-energy effective lagrangians. In particular they show how the exact moduli dependence of some F^4 terms in the eight-dimensional type II (4,0) orbifold is reproduced by the infinite sum of D-instanton contributions in the dual type I theory.

Dine [100] (1999) says that recently, a number of authors have challenged the conventional assumption that the string scale, Planck mass, and unification scale are roughly comparable. It has been suggested that the string scale could be as low as a TeV. The greatest obstacle to developing a string phenomenology is our lack of understanding of the ground state. He explains why the dynamics which determines this state is not likely to be accessible to any systematic approximation. He notes that the racetrack scheme, often cited as a counterexample, suffers from similar difficulties. He stresses that the weakness of the gauge couplings, the gauge hierarchy, and coupling unification suggest that it might be possible to extract some information in a systematic approximation. He reviews the ideas of Kähler stabilization, an attempt to reconcile these facts. He considers whether the system is likely to sit at extremes of the moduli space, as in recent proposals for a low string scale. Finally he discusses the idea of Maximally Enhanced Symmetry, a hypothesis which is technically natural, and hoped to be compatible with basic facts about cosmology, and potentially predictive.

Dvali and Tye [108] (1999) present a novel inflationary scenario in theories with low scale (TeV) quantum gravity, in which the standard model particles are localized on the branes whereas gravity propagates in the bulk of large extra dimensions. They say that this inflationary scenario is natural in the brane world picture. In the lowest energy state, a number of branes sit on top of each other (or at an orientifold plane), so the vacuum energy cancels out. In a cosmological setting, some of the branes "start out" relatively displaced in the extra dimensions and the resulting vacuum energy triggers the exponential growth of the 3 non-compact dimensions. They say that the number of e-foldings can be very large due to the very weak brane-brane interaction at large distances. In the effective four-dimensional field theory, the brane motion is described by a slowly rolling scalar field with an extremely flat plateau potential. They say that when branes approach each other to a critical distance, the potential becomes steep and inflation ends rapidly. Then the branes "collide" and oscillate about the equilibrium point, releasing energy mostly into radiation on the branes. See also §4.2.

King and Riotto [202] (1999) note that dilaton stabilization is usually considered to pose a serious obstacle to successful D-term inflation in superstring theories. They argue that the physics of gaugino condensation is likely to be modified during the inflationary phase in such a way as to enhance the gaugino condensation scale. This enables dilaton stabilization during inflation with the D-term still dominating the vacuum energy at the stable minimum. See also §4.2.

Blumenhagen *et al* [43] (2000) investigate the D-brane contents of asymmetric orbifolds. Using T-duality they find that the consistent description of open strings in asymmetric orbifolds requires to turn on background gauge fields on the D-branes. They derive the corresponding noncommutative geometry arising on such D-branes with mixed Neumann-Dirichlet boundary conditions directly by applying an asymmetric rotation to open strings with pure Dirichlet or Neumann boundary conditions. As a concrete application of their results they construct asymmetric type I vacua requiring open strings with mixed boundary conditions for tadpole cancellation.

Donets *et al* [375] (2000) discuss the brane vacuum as a chain of rotators by using the noncomuatative U(1) sigma model.

Ellis *et al* [115] (2000) note that classical superstring vacua have zero vacuum energy and are supersymmetric and Lorentz-invariant. They argue that all these properties may be destroyed when quantum aspects of the interactions between particles and non-perturbative vacuum fluctuations are considered. A toy calculation of string/D-brane interactions using a world-sheet approach indicates that quantum recoil effects - reflecting the gravitational back-reaction on spacetime foam due to the propagation of energetic particles - induce non-zero vacuum energy that is linked to supersymmetry breaking and breaks Lorentz invariance. This model of space-time foam also suggests the appearance of microscopic event horizons. Ellis *et al* [114] (2000) have not identified a vacuum with broken supersymmetry and zero vacuum energy.

Hata and Shinohara [165] (2000) note that tachyon condensation on a bosonic D-brane was recently demonstrated numerically in Witten's open string field theory with level truncation approximation. This non-perturbative vacuum, which is obtained by solving the equation of motion, has to satisfy furthermore the requirement of BRST invariance. This is indispensable in order for the theory around the non-perturbative vacuum to be consistent. They carry out the numerical analysis of the BRST invariance of the solution and find that it holds to a good accuracy. They also mention the zero-norm property of the solution. The observations in this paper are expected to give clues to the analytic expression of the vacuum solution.

Nudelman [394] (2000) considers certain linear objects, termed physical lines; and introduces initial assumptions concerning their properties. He investigates a closed physical line in the form of a circle called a J-string. He showns that this curve consists of indivisible line segments of length ℓ_Δ. It is assumed that a J-string has an angular momentum whose value is $\hbar$. It is then established that a J-string of radius R possesses a mass m_J, equal to $h/2\pi cR$, a corresponding energy, as well as a charge q_J, where $q_J = (hc/2\pi)^{1/2}$. He also establishes that $\ell_\Delta = 2\pi(hG/c^3)^{1/2}$, where c is the speed of light and G is the gravitational constant. Based upon investigation of the properties and characteristics of J-strings, he develops a method for the computation of the Planck length and mass (ℓ^*_P, m^*_P). Using the methods developed in the paper the values of ℓ^*_P and m^*_P are computed these values differ from the currently accepted ones.

Toms [340] (2000) considers the quantization of a scalar field in the five-dimensional model suggested by Randall and Sundrum. Using the Kaluza-Klein reduction of the scalar field, discussed by Goldberger and Wise, he sums the infinite tower of modes to find the vacuum energy density. Dimensional regularisation is used to compute the pole term needed for renormalisation, as well as the finite part of the energy density. He makes some comments concerning the possible self-consistent determination of the radius.

2.9 Lattice Models.

Unlike QFT's which are defined at each point of spacetime, lattice models are only defined at, a usually finite number of, discrete points. A finite number of points might result in less of the infinite objects which occur in QFT's. Luo [232] (1998) investigate the properties of lattice QCD, see §2.5. Some recent work on vacua occurring in lattice models is listed below.

Hollenberg [177] (1994) computes the vacuum energy density for a SU(2) lattice gauge theory and gets results which might apply to beyond the reach of the strong to weak transition point $g_c^2 \approx 2.0$.

Bock *et al* [44] (1995) note that lattice proposals for a nonperturbative formulation of the Standard Model easily lead to a global U(1) symmetry corresponding to exactly conserved fermion number. The absence of an anomaly in the fermion current would then appear to inhibit anomalous processes, such as electroweak baryogenesis in the early universe. One way to circumvent this problem is to formulate the theory such that this U(1) symmetry is explicitly broken.

Adam [3] (1997) gives a detailed discussion of the mass perturbation theory of the massive Schwinger model. After discussing some general features and briefly reviewing the exact solution of the massless case, he computes the vacuum energy density of the massive model and some related quantities. He derives the Feynman rules of mass perturbation theory and discuss the exact n-point functions with the help of the Dyson-Schwinger equations. Furthermore he identifies the stable and unstable bound states of the theory and computes some bound-state masses and decay widths. Finally he discuss scattering processes, where he claims that the resonances and particle production thresholds of the model are properly taken into account by his methods.

Aroca [19] (1999) studies the Schwinger model in a finite lattice by means of the P-representation. The evaluate the vacuum energy, mass gap and chiral condensate showing good agreement with the expected values in the continuum limit.

Cea and Cosmai [71] (1999) study the vacuum dynamics of SU(2) lattice gauge theory is studied by means of a gauge-invariant effective action defined using the lattice Schrödinger functional. they perform numerical simulations both at zero and finite temperature. They probe the vacuum using an external constant Abelian chromomagnetic field. Their results suggest that at zero temperature the external field is screened in the continuum limit. On the other hand at finite temperature they say that it seems that confinement is restored by increasing the strength of the applied field.

Maniadas *et al* [239] (1999) use a collective coordinate approach to investigate corpuscular properties of breathers in nonlinear lattice systems. They calculate the breather internal energy and inertial mass and use them to analyze the reaction pathways of breathers with kinks that are performed in the lattice. They find that there is an effective kink breather interaction potential, that, under some circumstances, is attractive and has a double well shape. Furthermore, they find that in some cases the internal energy of a moving breather can be realized during the reaction with the kink and subsequently transformed to kink translational energy. These breather properties seem to be model independently. See also §2.14.

Actor *et al* [2] (2000) present a Hamiltonian lattice formulation of static Casimir systems at a level of generality appropriate for an introductory investigation. Background structure - represented by a lattice potential V(x) - is introduced along one spatial direction with translation invariance in all other spatial directions. After some general analysis they analyze two specific finite one dimensional lattice QFT systems. See also §3.

2.10 Symmetry Breaking.

One of the outstanding problems of particle physics is whether the Higgs field exists; this hypothetical field is conjectured to make non-Abelian gauge fields describe massive particles as needed by the standard particle physics model, see §2.1. Mass comes from the vacuum expectation value of the field differing from zero, in this manner symmetry breaking can be thought of as a vacuum energy effect. Baum [30] (1994) discusses a positive definite action which leads to no cosmological constant and breaks symmetry. The way that symmetry is broken invokes properties of the vacuum, see equation 2.34. Vacuum energy essentially "shifts" the value of the scalar field: and this shift corresponds to mass. For purposes of illustration symmetry breaking in scalar electrodynamics (SEL) is presented below; although because electromagnetism is massless and the only theory with just one vector A_a symmetry breaking is not physically realize here. The scalar electrodynamic Lagrangian [174], [181]p.68, [167]p.699 is

$$\mathcal{L}_{sel} = -D_a\psi D^a\bar{\psi} - V(\psi\bar{\psi}) - \frac{1}{4}F^2, \tag{2.21}$$

where the covariant derivative is

$$D_a\psi = \partial_a\psi + ieA_a, \tag{2.22}$$

and $D_a\bar{\psi} = \bar{D_a}\psi$; and so is related to the lagrangian in 2.1 §2.1 by changing the partial derivatives to covariant derivatives involving the vector potential A_a. The variation of the corresponding action with respect to A_a, ψ, and $\bar{\psi}$ are given by

$$\begin{aligned}
\frac{\delta I}{\delta A_c} &= F^{ab}_{..;b} + ie(\psi D^a\bar{\psi} - \bar{\psi}D^a\psi) \\
\frac{\delta I}{\delta\psi} &= (D_aD^a - V')\bar{\psi}, \quad V' = \frac{dV}{d(\psi\bar{\psi})},
\end{aligned} \tag{2.23}$$

and its complex conjugate. Variations of the metric give the stress

$$T_{ab} = 2D_{(a}\psi D_{b)}\bar{\psi} + F_{ac}F_{b.}^{\ \ c} + g_{ab}\mathcal{L}. \tag{2.24}$$

The complex scalar field can be put in "polar" form by defining

$$\psi = \rho\exp(i\nu), \tag{2.25}$$

giving the Lagrangian

$$\mathcal{L} = \rho_a^2 + (\mathcal{D}_a\nu)^2 - V(\rho^2) - \frac{1}{4}F^2, \tag{2.26}$$

where

$$\mathcal{D}_a\nu = \rho(\nu_a + eA_a). \tag{2.27}$$

The variations of the corresponding action with respect to A_a, ρ, and ν are given by

$$\begin{aligned} \frac{\delta I}{\delta A_a} &= F^{ab}_{..;b} + 2e\rho\mathcal{D}^a\nu, \\ \frac{\delta I}{\delta \rho} &= 2\left(\Box + (\nu_a + eA_a)^2 + V'\right), \\ \frac{\delta I}{\delta \nu} &= 2\left(\Box\nu + eA^a_{.a;a}\right). \end{aligned} \tag{2.28}$$

Variation of the metric gives the stress

$$T_{ab} = 2\rho_a\rho_b + 2\mathcal{D}_a\nu\mathcal{D}_b\nu + F_{ac}F_{b.}^{\ c} + g_{ab}\mathcal{L}. \tag{2.29}$$

Defining

$$B_a = A_a + \nu_a/e, \tag{2.30}$$

ν is absorbed to give Lagrangian

$$\mathcal{L} = \rho_a^2 + \rho^2 e^2 B_a^2 - V(\rho^2) - \frac{1}{4}F^2, \tag{2.31}$$

which does not contain ν; equation 2.30 is a gauge transformation when there are no discontinuities in ν, i.e. $\nu_{;[ab]} = 0$.

The requirement that the corresponding quantum theory is renormizable restricts the potential to the form

$$V(\rho^2) = m^2\rho^2 + \lambda\rho^4. \tag{2.32}$$

The ground state is when there is a minimum, for $m^2, \lambda > 0$ this is $\rho = 0$, but for $m^2 < 0, \lambda > 0$ this is

$$\rho^2 = \frac{-m^2}{2\lambda} = a^2, \tag{2.33}$$

thus the vacuum energy is

$$< 0|\rho|0 >= a. \tag{2.34}$$

To transform the Lagrangian 2.31 to take this into account substitute

$$\rho \to \rho' = \rho + a, \tag{2.35}$$

to give

$$\mathcal{L} = \rho_a^2 + (\rho + a)^2 e^2 B_a^2 - V\left((\rho + a)^2\right) - \frac{1}{4}F^2. \tag{2.36}$$

Now apparently the vector field has a mass m from the $a^2e^2B_a^2$ term it is given by $m = ae$. The cross term $2a\rho e^2 B_a^2$ is ignored.

There are well known techniques by which stresses involving scalar fields can be rewritten as fluids, so that it is possible to re-write the standard model with fluids instead of Higgs scalars. It is preferable to use fluids rather than

Higgs scalar fields for *two* reasons. The *first* is because Higgs scalar fields are *ad hoc* whereas fluids might arise from the statistical properties of the non-Abelian gauge fields: furthermore taking an extreme view of the principle of equivalence Roberts [290] (1989) suggests that Higgs scalars cannot be fundamental. The *second* is that perfect fluids are gauge systems and thus all matter under consideration is part of a gauge system. My *first* attempt at using fluids for symmetry breaking Roberts [290] (1989) was essentially to deploy rewriting procedures to convert the scalar fields to fluids; the drawback of this approach is the fluids that result are somewhat unphysical, however an advantage is that symmetry breaking occurs with a change of state of the fluid. My *second* attempt Roberts [295] (1997) used the decomposition of a perfect fluid vector into several scalar parts called vector potentials and identifying one of these with the radial Higgs scalar ρ, see equation 2.35. The gauge fields are then introduced by using the usual covariant substitutions for the partial derivatives of the scalars, for example $\phi_a = \partial_a\phi \rightarrow \nabla_a\phi = \partial_a\phi + ie\phi A_a$, and calling the resulting fluid the COVARIANTLY INTERACTING FLUID. This results in an elegant extension of standard Higgs symmetry breaking, but with some additional parameters present. Previous work on the vector potentials shows that some of these have a thermodynamic interpretation, it is hoped that this is inherited in the fluid symmetry breaking models. The additional parameters can be partially studied with the help of an explicit Lagrangian Roberts [299] (1999). Both of my approaches have been restricted to Abelian gauge fields.

Some recent papers on symmetry breaking include those below. Coleman and Weinberg [84] §2 (1973) show that functional methods allow the definition of an effective potential, the minimum of which, without any approximation, gives the true vacuum states of a theory. Kujat and Scherrer [211] discuss the cosmological implications of a time dependent Higgs field, see §4.5 above.

Kounnas *et al* [204] (1994) note that in the minimal supersymmetric standard model (MSSM), the scale M_{SUSY} of soft supersymmetry breaking is usually *assumed* to be of the order of the electroweak scale. They reconsider here the possibility of treating M_{SUSY} as a dynamical variable. Its expectation value should be determined by minimizing the vacuum energy, after including MSSM quantum corrections. They point out the crucial role of the cosmological term for a dynamical generation of the desired hierarchies $m_Z, M_{SUSY} << M_P$, here yet again the cosmological term is related to the vacuum energy. Inspired by four-dimensional superstring models, they also consider the Yukawa couplings as dynamical variables. They find that the top Yukawa coupling is attracted close to its effective infrared fixed point, corresponding to a top-quark mass in the experimentally allowed range. As an illustrative example, they present the results of explicit calculations for a special case of the MSSM.

Ferrara *et al* [122] (1995) show that the minimal Higgs sector of a generic N=2 supergravity theory with unbroken N=1 supersymmetry must contain a Higgs hypermultiplet and a vector multiplet. When the multiplets parameterize the quaternionic manifold SO(4,1)/SO(4), and the special Kähler manifold SU(1,1)/U(1), respectively, a vanishing vacuum energy with a sliding massive spin 3/2 multiplet is obtained. Potential applications to N=2 low energy effective actions of superstrings are briefly discussed.

Boyanovsky *et al* [51] (1996) analyze the phenomenon of preheating,i.e. explosive particle production due to parametric amplification of quantum fluctuations in the unbroken case, or spinodal instabilities in the broken phase, using the Minkowski space $O(N)$ vector model in the large N limit to study the non-perturbative issues involved. They give analytic results for weak couplings and times short compared to the time at which the fluctuations become of the same order as the tree level, as well as numerical results including the full backreaction. In the case where the symmetry is unbroken, the analytic results agree spectacularly well with the numerical ones in their common domain of validity. In the broken symmetry case, slow roll initial conditions from the unstable minimum at the origin, give rise to a new and unexpected phenomenon: the dynamical relaxation of the vacuum energy. That is, particles are abundantly produced at the expense of the quantum vacuum energy while the zero mode comes back to almost its initial value. In both cases they obtain analytically and numerically the equation of state which turns to be written in terms of an effective polytropic index that interpolates between vacuum and radiation-like domination. They find that simplified analysis based on harmonic behavior of the zero mode, giving rise to a Mathieu equation for the non-zero modes misses important physics. Furthermore, they claim that analysis that do not include the full backreaction do not conserve energy, resulting in unbound particle production. Their results do not support the recent claim of symmetry restoration by non-equilibrium fluctuations. Finally estimates of the reheating temperature are given, as well as a discussion of the inconsistency of a kinetic approach to thermalization when a non-perturbatively large number of particles is created.

Das and Pernice [91] (1996) propose a new mechanism for symmetry breaking which naturally avoids the constraints following from the usual theorems of symmetry breaking. In the context of super-symmetry, for example, the breaking may be consistent with a vanishing vacuum energy. A 2+1 dimensional super-symmetric gauge field theory is explicitly shown to break super-symmetry through this mechanism while maintaining a zero vacuum energy. This mechanism may provide a solution to two long standing problems, namely, dynamical super-symmetry breaking and the cosmological constant problem, see §4.1.

Axenides and Perivolaropoulos [20] (1997) demonstrate that field theories involving explicit breaking of continuous symmetries, incorporate two generic classes of topological defects each of which is stable for a particular range of parameters. The first class includes defects of the usual type where the symmetry gets restored in the core and vacuum energy gets trapped there. However they show that these defect solutions become unstable for certain ranges of parameters and decay not to the vacuum but to another type of stable defect where the symmetry in not restored in the core. In the wall case, initially spherical, bubble-like configurations are simulated by them numerically and shown to evolve generically towards a planar collapse. In the string case, the decay of the symmetric core vortex resembles the decay of a semilocal string to a skyrmion with the important difference that while the skyrmion is unstable and decays to the vacuum, the resulting non-symmetric vortex is topologically stable.

Natale and da Silva [256] (1997) discuss how to dynamically break symmetry.

Lepora and Kibble [220] (1999) analyze symmetry breaking in the Weinberg-Salam model paying particular attention to the underlying geometry of the theory. In this context they find two natural metrics upon the vacuum manifold: an isotropic metric associated with the scalar sector, and a squashed metric associated with the gauge sector. Physically, the interplay between these metrics gives rise to many of the non-perturbative features of Weinberg-Salam theory.

Foot *et al* [128] (2000) note that if the Lagrangian of nature respects parity invariance then there are two distinct possibilities: either parity is unbroken by the vacuum or it is spontaneously broken. They examine the two simplest phenomenologically consistent gauge models which have unbroken and spontaneously broken parity symmetries, respectively. These two models have a Lagrangian of the same form, but a different parameter range is chosen in the Higgs potential. They both predict the existence of dark matter and can explain the MACHO events. However, the models predict quite different neutrino physics. Although both have light mirror (effectively sterile) neutrinos, the ordinary-mirror neutrino mixing angles are unobservably tiny in the broken parity case. The minimal broken parity model therefore cannot simultaneously explain the solar, atmospheric and LSND data. By contrast, the unbroken parity version can explain all of the neutrino anomalies. Furthermore, they argue that the unbroken case provides the most natural explanation of the neutrino physics anomalies (irrespective of whether evidence from the LSND experiment is included) because of its characteristic maximal mixing prediction.

2.11 Φ^4 Theory and the Renormalization Group.

If one starts with the view that a quantum description of reality is correct then one must assume that a quantum field theory (QFT) description of reality will be correct, as opposed to a classical (no Planck's constant $\hbar$) field theory description of reality. There are only a limited number of classical field theories that can be quantized to produce a QFT, the quantization schemes do not necessarily give a unique QFT. The reasons that classical field theories cannot be quantized are either technical or that they give infinite results or both. There might be classical theories which for a variety of reasons there is no corresponding quantum theory, starting with the view that only a quantum description of reality is fundamentally correct suggests that such theories are of no fundamental importance. The simplest classical field theories involve only scalar fields and those that give finite QFT's have an action 2.1 which has lagrangian 2.2 and potential 2.3. In the potential the ϕ^3 term does not seem to correspond to anything, thus ϕ^4 theories are picked out as well defined scalar QFT's. Lagrangians of this form are used in symmetry breaking, see the previous section §2.10. Langfeld *et al* [216] (1995) discuss Φ^4 theory and the Casimir effect. Two recent papers on Φ^4 theory are those below.

Borsanyi *et al* [51] (2000) study thermalisation of configurations with initial white noise power spectrum in numerical simulations of a one-component Φ^4 theory in 2+1 dimensions, coupled to a small amplitude homogeneous external field. The study is performed for energy densities corresponding to the broken symmetry phase of the system in equilibrium. The effective equation of the order parameter motion is reconstructed from its trajectory which starts from an initial value near the metastable point and ends in the stable ground state. They say that this phenomenological theory quantitatively accounts for the decay of the false vacuum. The large amplitude transition of the order parameter between the two minima displays characteristics reflecting the dynamical effect of the Maxwell construction.

Schützhold *et al* [309] (2000) show that the massless neutral $\lambda\Phi^4$-theory does not possess a unique vacuum. Based on the Wightman axioms the nonexistence of a state which preserves Poincaré and scale invariance is demonstrated non-perturbatively for a non-vanishing self-interaction. They conclude that it is necessary to break the scale invariance in order to define a vacuum state. They derive the renormalized vacuum expectation value of the energy-momentum tensor as well as the ϕ-onic and scalar condensate from the two-point Wightman function employing the point-splitting technique. They point out possible implications to other self-interacting field theories and to different approaches in quantum field theory.

Kastening [406] (1992) studies the renormalization group running of the effective potential which includes a renormalization group running of the vacuum energy. This is one of the motivations for the computations in the four- and five-loop papers of his [407, 192] discussed below.

Renormalization is a process by which the infinities of QFT's can sometimes be removed, often such a process has some freedom which forms a "group". Some recent work on the vacuum and this includes Christie *et al* [79] (1999) who use asymptotic Pade-approximant methods to estimate from prior orders of perturbation theory the five-loop contributions to the coupling-constant β-function β_g, the anomalous mass dimension γ_m, the vacuum-energy β-function β_v, and the anomalous dimension γ_2 of the scalar field propagator, within the context of massive N-component ϕ^4 scalar field theory. They compare these estimates with explicit calculations of the five-loop contributions to β_g, γ_m, β_v, and are seen to be respectively within 5%, 18%, and 27% of their true values for N between 1 and 5. They then extend asymptotic Pade-approximant methods to predict the presently unknown six-loop contributions to β_g, γ_m, and β_v. These predictions, as well as the six-loop prediction for γ_2, provide a test of asymptotic Pade-approximant methods against future calculations.

Kastening [192] (1997) uses dimensional regularization in conjunction with the MSbar scheme to analytically compute the β-function of the vacuum energy density at the five-loop level in O(N)-symmetric ϕ^4 theory, see §2.11. The result for the case of a cubic anisotropy is also given. It is pointed out how to also obtain the beta function of the coupling and the gamma function of the mass from vacuum graphs. They say that this method may be easier than traditional approaches. The four loop equivalent of this, where the conventions are set is, Kastening [407].

2.12 Instantons and θ-Vacua.

Instantons are defined by Kaku [187] §16.6 (1993) as finite action solutions to equations of motion with positive definite metric, they allow tunneling between different vacua because they connect vacua at $x_4 \to \pm\infty$ thus the naive vacuum is unstable. The instanton allows tunneling between all possible vacua labeled by winding number n. Thus the true vacuum must be a superposition of the various vacua $|n>$ belonging to some different homotopy class. The effect of a gauge transformation Ω_1 is to shift the winding number n by one:

$$\Omega_1 : |n> \to |n+1> . \tag{2.37}$$

Since the effect of Ω_1 on the true vacuum can change it only by an overall phase factor, this fixes the coefficients of the various vacua $|n>$ within the true vacuum. This fixes the coefficients of $|n>$ as follows:

$$|vac>_\theta = \sum_{n=-\infty}^{\infty} e^{in\theta}|n> . \tag{2.38}$$

Some recent work on θ-vacua includes Yi [363] (1994) finds that the sign of the vacuum energy density effects the geometry of dilatonic extremal black holes.

Etesi [119] (2000) studies the existence of θ-vacuum states in Yang-Mills theories defined over asymptotically flat, stationary spacetimes taking into account not only the topology but the complicated causal structure of these spacetimes. By a result of Chrusciel and Wald, apparently causality makes all vacuum states, seen by a distant observer, homotopically equivalent making the introduction of θ-terms unnecessary in causally effective Lagrangians. He claims that a more careful study shows that certain twisted classical vacuum states survive even in this case eventually leading to the conclusion that the concept of "θ-vacua" is meaningful in the case of general Yang-Mills theories. He gives a classification of these vacuum states based on Isham's results showing that the Yang-Mills vacuum has the same complexity as in the flat Minkowskian case hence the general CP-problem is not more complicated than the well-known flat one.

Zhitnitsky [366] (2000) notes that it has been recently argued that an arbitrary induced θ-vacuum state could be created in the heavy ion collisions, similar to the creation of the disoriented chiral condensate with an arbitrary isospin direction. It should be a large domain with a wrong θ orientation which will mimic the physics of the world when the fundamental θ is non-zero. He suggest a few simple observables which can hopefully be measured on an event by event basis at RHIC, and which uniquely determine whether the induced θ-vacuum state is created.

Halpern and Zhitnitsky [164] (1998) suggest that the topological susceptibility in gluodynamics can be found in terms of the gluon condensate using renormalizability and heavy fermion representation of the anomaly. Analogous relations can be also obtained for other zero momentum correlation functions involving the topological density operator. Using these relations, they find the θ

dependence of the condensates $< GG >, < G\tilde{G} >$ and of the partition function for small θ and an arbitrary number of colours.

Some recent work on instantons includes Kochelev [201] (1995) who shows that specific properties of the instanton induced interaction between quarks leads to the anomalous violation of the OZI-rule in the $N\bar{N} \to \Phi\Phi$, $N\bar{N} \to \Phi\gamma$ reactions. In the framework of instanton model of the QCD vacuum, the energy dependence of the cross sections of these reactions is calculated.

Yung [364] (1995) considers instanton dynamics in the broken phase of the topological σ model with the black hole metric of the target space. It has been shown before that this model is in the phase with BRST-symmetry broken. In particular he says that vacuum energy is non-zero and correlation functions of observables show the coordinate dependence. However he says that these quantities turned out to be infrared (IR) divergent. Yung shows that IR divergences disappear after the sum over an arbitrary number of additional instanton-anti-instanton pairs is performed. The model appears to be equivalent to Coulomb gas/Sine Gordon system.

Zhitnitsky [366] (2000) notes that it has been recently argued that an arbitrary induced θ-vacuum state could be created in the heavy ion collisions, similar to the creation of the disoriented chiral condensate with an arbitrary isospin direction. It should be a large domain with a wrong θ orientation which will mimic the physics of the world when the fundamental θ is non-zero. He suggest a few simple observables which can hopefully be measured on an event by event basis at RHIC, and which uniquely determine whether the induced θ-vacuum state is created.

2.13 Solitons, Integrable Models and Magnetic Vortices.

Solitons are solutions to differential equations which have well defined properties, such as their energy does not disperse, see Roberts [284] (1985). Roughly speaking, a soliton is a stable, localized, finite energy solution to a classical equation. The idea of a soliton has been extended to curved spacetimes and reasonable necessary conditions for a classical solution to be a soliton are:

1. the solution is asymptotically flat and admits a one-dimensional symmetry group whose trajectories are time-like;

2. the energy density is localized and the total energy is finite;

3. the solution is classically stable;

4. the solution is quantum mechanically stable.

In gravitational theory usually just Kerr-Newman solutions are considered as possible candidates for solitons. However, generic Kerr-Newmann solutions have horizons and so are considered quantum mechanically unstable because of the Hawking effect. Naked Kerr-Newmann solutions have negative energy and extreme Kerr-Newmann solutions are destroyed by the fermionic vacuum; the static spherically symmetric Einstein scalar fields are discussed in Roberts [284] (1985) do not possess event horizons. All the solutions discussed are asymptotically flat and static and so satisfy 1. The qualitative features of these solutions, such as no event horizon, remain, no matter how small the scalar field is and so they can be made to have a total energy arbitrary similar to the Schwarzschild solution, and in any case their Tolman energy is positive and they obey reasonable energy conditions and so 2 is satisfied. The solutions are static solutions of the field equations for which perturbations do not change the qualitative features and so 3 is satisfied. It has been suggested that curvature always produces particles Roberts [285] (1986). In this case there would be no quantum mechanical stability. However, it is usually thought that only spacetimes with event horizons produce particles, but Einstein scalar solutions do not have event horizons and so satisfy 4. Whether there is a mechanism by which the Einstein scalar solutions can excite the vacuum remains to be investigated.

Some recent work on solitons and the vacuum include Müller-Kirsten *et al* [254] (1998) who suggest the the shift in vacuum energy in the O(3) Skyrme model is due to instantons and Moss [252] (1999) who relates the the quantum properties of solitons at one loop to phase shifts of waves on the soliton background. These can be combined with heat kernel methods to calculate various parameters. The vacuum energy of a CP(1) soliton in 2+1 dimensions is calculated as an example.

Some two dimensional theories can be solved completely and are called integral models. Some recent work on the vacuum in integrable models include the paper of Delfino *et al* [94] (1996) who approaches the study of non–integrable models of two–dimensional quantum field theory as perturbations of the integrable ones. By exploiting the knowledge of the exact S-matrix and form factors of the integrable field theories they obtain the first order corrections to the mass ratios, the vacuum energy density and the S-matrix of the non-integrable theories. As interesting applications of the formalism Delfino *et al* [94] study the scaling region of the Ising model in an external magnetic field at $T \sim T_c$ and the scaling region around the minimal model $M_{2,7}$. For these models they observe a remarkable agreement between the theoretical predictions and the data extracted by a numerical diagonalization of their Hamiltonian.

Babansky,A.Yu and Sitenko,Ya,A. [23] (1999) discuss vacuum energy induced by a singular magnetic vortex. Langfeld *et al* [215] (1998) note that the magnetic vortices which arise in SU(2) lattice gauge theory in center projection are visualized for a given time slice. They establish that the number of vortices piercing a given 2-dimensional sheet is a renormalization group invariant and therefore physical quantity. They find that roughly 2 vortices pierce an area of 1 fm^2.

2.14 Topological Quantum Field Theory.

There are a variety of objects which can be "quantized", i.e. starting with a classical (no $\hbar$) system one can produce a quantum system which corresponds to it. Topology is the study of shapes and one can ask whether shapes can be quantized; in particular Witten [360] (1988) notes that topological quantum field theory has the formal structure of a quantum theory (e.g. dealing with probabilities), but that the information they produce is purely topological (e.g. information about the nature of a knot). Topological QFT's have been recently reviewed by Schwarz [313] (2000). That Casimir energy might be topological is discussed in Williams [357] (1997).

2.15 SAZ Approach.

A way of approaching the quantum vacuum is the Sakarov [303](1967) -Adler [6] (1983) -Zee [365] (1983) approach (explained for example in Misner, Thorne and Wheeler [247] page 426 (1970)) where the quantum vacuum again induces a cosmological constant. Their approaches are covariant and so are compatible with general relativity, no doubt there are approaches that are not covariant. Because of covariance the quantum vacuum problem and the inertia problem become separate issues. Pollock and Dahder [276] (1989) discuss how the SAZ approach fits in with inflation.

Belgiorno and Liberati [36] (1997) show an analogy between the subtraction procedure in the Gibbons-Hawking Euclidean path integral approach to horizon's thermodynamics and the Casimir effect. Then they conjecture about a possible Casimir nature of the Gibbons-Hawking subtraction is made in the framework of Sakharov's induced gravity. In this framework it appears that the degrees of freedom involved in the Bekenstein-Hawking entropy are naturally identified with zero–point modes of the matter fields. They sketch some consequences of this view.

Consoli [86] (2000) notes that the basic idea that gravity can be a long-wavelength effect *induced* by the peculiar ground state of an underlying quantum field theory leads to consider the implications of spontaneous symmetry breaking through an elementary scalar field. He point out that Bose-Einstein condensation implies the existence of long-range order and of a gap-less mode of the (singlet) Higgs-field. This gives rise to a $1/r$ potential and couples with infinitesimal strength to the inertial mass of known particles. If this is interpreted as the origin of Newtonian gravity one finds a natural solution of the hierarchy problem. As in any theory incorporating the equivalence principle, the classical tests in weak gravitational fields are fulfilled as in general relativity. On the other hand, our picture suggests that Einstein general relativity may represent the weak field approximation of a theory generated from flat space with a sequence of conformal transformations. This explains naturally the absence of a *large* cosmological constant from symmetry breaking. Finally, one also predicts new phenomena that have no counterpart in Einstein theory such as typical 'fifth force' deviations below the centimeter scale or further modifications at distances

10^{17} cm in connection with the Pioneer anomaly and the mass discrepancy in galactic systems.

Guendelman and Portnoy [158] (1999) consider a model of an elementary particle as a 2 + 1 dimensional brane evolving in a 3 + 1 dimensional space. Introducing gauge fields that live in the brane as well as normal surface tension can lead to a stable "elementary particle" configuration. Considering the possibility of non vanishing vacuum energy inside the bubble leads, when gravitational effects are considered, to the possibility of a quantum decay of such "elementary particle" into an infinite universe. Some remarkable features of the quantum mechanics of this process are discussed, in particular the relation between possible boundary conditions and the question of instability towards Universe formation is analyzed.

Guendelman [155] (1999) discusses the possibility of mass in the context of scale-invariant, generally covariant theories. Scale invariance is considered in the context of a gravitational theory where the action, in the first order formalism, is of the form $S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x$ where Φ is a density built out of degrees of freedom independent of the metric. For global scale invariance, a "dilaton" ϕ has to be introduced, with non-trivial potentials $V(\phi) = f_1 e^{\alpha\phi}$ in L_1 and $U(\phi) = f_2 e^{2\alpha\phi}$ in L_2. This leads to non-trivial mass generation and a potential for ϕ which is interesting for inflation. The model can be connected to the induced gravity model of Zee [365] (1983), which is a successful model of inflation. Models of the present universe and a natural transition from inflation to a slowly accelerated universe at late times are discussed.

2.16 Superfluids and Condensed Matter.

Fields are not the only objects which one can think of as occupying spacetime, there are also fluids; when there is no equation of state specified they are more general than fields. Quantum fluids as usually understood are a limited class of object used to discuss low temperature phenomena. There are many analogies between such "superfluids" and QFT's, see Volkovik [350] (2000). So far there has been no contact between low temperature superfluids and the quantized fluids discussed by Roberts [298] (1999). Some recent work on vacua and superfluids includes Fischer *et al* [126] (1993) who find that all systems under their investigation exhibit an occupied alkali-mixture state near the Fermi energy as well as a series of unoccupied states converging towards the vacuum energy, which are identified as image-potential states; thus in this case vacuum energy is associated with a state of the system. Duan [106] (1993) uses the concept of finite compressibility of a Fermi superfluid to reconsider the problem of inertial mass of vortex lines in both neutral and charge superfluids at zero temperature $T = 0$.

Chapline [74] (1998) shows that a simple model for 4-dimensional quantum gravity based on a 3-dimensional generalization of anyon superconductivity can be regarded as a discrete form of Polyakov's string theory. This suggests that there is a universal negative pressure that is on the order of the string tension divided by the square of the Robertson-Walker scale factor. This is in accord with recent observations of the brightness of distant supernovae, which suggest that at the present time there is a vacuum energy whose magnitude is close to the mass density of an Einstein-de Sitter universe.

Volovik [350] (2000) notes that superfluid 3He-A gives an example of how chirality, Weyl fermions, gauge fields and gravity appear in low energy corner together with corresponding symmetries, including Lorentz symmetry and local SU(N). This supports idea that quantum field theory Standard Model or GUT is effective theory describing low-energy phenomena. *Seven* reasons for this are: *firstly* momentum space topology of fermionic vacuum provides topological stability of universality class of systems, where above properties appear. *Secondly* BCS scheme for 3He-A incorporates both "relativistic" infrared regime and ultraviolet "transPlanckian" range: subtle issues of cut-off in quantum field theory and anomalies can be resolved on physical grounds. This allows to separate "renormalizable" terms in action, treated by effective theory, from those obtained only in "transPlanckian" physics. *Thirdly* energy density of superfluid vacuum within effective theory is E^4_{Planck}. Stability analysis of ground state beyond effective theory leads to exact nullification of vacuum energy: equilibrium vacuum is not gravitating. In nonequilibrium, vacuum energy is of order energy density of matter. *Fourthly* 3He-A provides experimental prove for anomalous nucleation of fermionic charge according to Adler-Bell-Jackiw. *Fifthly* helical instability in 3He-A is described by the same equations as formation of magnetic field by right electrons in Joyce-Shaposhnikov scenario. *Sixthly* macroscopic parity violating effect and angular momentum paradox are both described by axial gravitational Chern-Simons action. *Seventhly* high energy dispersion of quasiparticle spectrum allows treatment of problems of vacuum in presence of an event horizon.

Elstov *et al* [117] (2000) note that spin-mass vortices have been observed to form in rotating superfluid 3He-B following the absorption of a thermal neutron and a rapid transition from the normal to superfluid state. The spin-mass vortex is a composite defect which consists of a planar soliton (wall) which terminates on a linear core (string). This observation fits well within the framework of a cosmological scenario for defect formation, known as the Kibble-Zurek mechanism. It suggests that in the early Universe analogous cosmological defects might have formed.

Fischer *et al* [126] (1993) note that high-resolution photoemission and two-photon photoemission spectroscopy have been used for a comparative study of the electronic structure of a single layer of Na and K on $Cu(111)$, $Co(0001)$ and $Fe(110)$. All of the systems under investigation exhibit an occupied alkali-induced state near the Fermi energy as well as a series of unoccupied states converging toward the vacuum energy, which are identified as image-potential states....

3 The Casimir and Related Effects.

3.1 Introduction.

Lamoreaux [213] (1999) provides an introductory guide to the literature on the Casimir effect with 368 references; he points out that there are over a $1,000$ references on this subject and the number doubles every five years; in part this is due to what is called the "Casimir effect" changing, contemporary usage does not necessarily require fixed plate boundaries. The Casimir [69] (1948) effect can be characterized as a force that arises when some of the normal modes of a zero rest-mass field such as the electromagnetic field are excluded by boundary conditions. If one places in Minkowski space-time two parallel flat perfect conductors, then the boundary conditions on the conductors ensure that normal modes whose wavelength exceeds the spacing of the conductors are excluded. If now the conductors are moved slightly apart, new normal modes are permitted and the zero-point energy is increased. Work must be done to achieve this energy increase, and so there must be an attractive force between the plates. This force has been measured, and the zero-point calculation verified. This agreement with experiment is important, since it shows that calculations with zero-point energy of a continuous field does have some correspondence with reality, although the total energy associated with modes of arbitrarily high frequency is infinite. The Casimir effect shows that finite differences between different configurations of infinite energy do have physical reality. Other example of this principle for zero-point effects is the Lamb shift and the Toll-Scharnhorst effect, see §2.3 above.

3.2 History of the Casimir Effect.

This subsection largely follows Fulling [134] (1989). Casimir [69] (1948) tried to calculate the van der Waals force between two polarizable atoms. The charge fluctuations in one atom can create an electric field capable of polarizing the charge in the other atom, so that there is a net force between them. To simplify the analysis, Casimir [69] (1948) considered a similar force between an atom and a conducting plate. From there he was led to the problem of two parallel plates. It was found that to get agreement with experiment in any of these problems, it was necessary to take into account the finiteness of the speed of light, which implies that the influence of charge fluctuations in one part of the system reaches the other part only after a delay. This means that the energy stored in the electromagnetic field passing between the two bodies must be taken into account. In fact, it turned out that the *long range limit* of the force can be associated *entirely* with the change in the energy of the field as the distance between the bodies varies; the charge fluctuations faded into the background of the analysis.

Formally the field energy is a sum over the divergent modes of the field. Casimir calculated the energy change ΔE, by inserting an *ad hoc* convergence factor, usually $e^{-\omega_n/\Lambda}$ with Λ a large constant, to make the two energies finite.

Their difference, ΔE, has a finite limit as $\Lambda \to \infty$. Replacing ω_n by n in the exponential gives a different answer. This exponential cutoff is algebraically equivalent to an analytic continuation of the point separation along the time axis: as can be seen by taking $\Lambda = i(t - t')$.

The physical interpretation offered for this procedure was this - no physical conductor is perfect at arbitrarily high frequencies; a very high-frequency wave should hardly notice the presence of the plates at all. Therefore, in a real experiment ΔE *must* be finite. An integral or sum defining it must have an effective cutoff depending on the detailed physics of the materials; $e^{-\omega_n/\Lambda}$ is a plausible model. The fact that the result is independent of Λ in the limit of large Λ suggests that this model is roughly correct and a more detailed model is unnecessary for a basic understanding.

This paragraph largely follows Lamoreaux [214] (1999). For conducting parallel flat plates separated by distance r, this force per unit area A has the magnitude Casimir [69] (1948)

$$\frac{F(r)}{A} = \frac{\pi^2}{240}\frac{\hbar c}{r^4} \approx \frac{1.3 \times 10^{-2}}{r^4}\text{dyn}(\mu\text{m})^4.\text{cm.}^2. \tag{3.1}$$

Casimir derived this relation by considering the electromagnetic mode structure between the two parallel plates of infinite extent, as compared to the mode structure when the plates are infinitely far apart, and by assigning a zero-point energy of $1/2\hbar\omega$ to each electromagnetic mode (photon). The change in total energy density between the plates, as compared to free space, as a function of separation r, leads to the force of attraction. This result is remarkable partly because it was one of the first predictions of a physical consequence directly due to zero-point fluctuations, and was contemporary with, but independent of, Bethe's treatment of the Lamb shift. Lifshitz [226] (1956) first developed the theory for the attractive force between two plane surfaces made of a material with generalized susceptibility.

The attraction between smooth, very close surfaces was eventually demonstrated experimentally Tabor and Winterton [334] (1969). For technical reasons they used dielectric materials instead of conductors. The theory of the effect for dielectrics is more complicated than for conductors but leads to similar results.

The existence of pointlike charged particles has always been problematic within the framework of classical electrodynamics. In view of the negative vacuum energy in the configuration of conducting parallel plates, Casimir proposed a model of the electron as a charged sphere with properties like those of a microscopic conductor. The hope was that an attractive force arising from the dependence of the vacuum energy on the sphere's radius would balance the electrostatic self-repulsion of the charge distribution, thereby holding the electron together stably. Boyer [52] (1970) and Davies [93] (1972) succeeded in calculating a conducting spherical shell's vacuum energy. Boyer's results were fatal to the Casimir electron model; the Casimir force turned out to be *repulsive* in this case:

$$F = -\frac{\partial E}{\partial R} > 0. \tag{3.2}$$

Furthermore, he found that the energy associated with the presence of a spherical conducting shell is infinite; that is, the energy difference between a configuration with an inner shell and one without did not converge as the cutoff was removed. In the presence of a *curved* conducting boundary the electromagnetic field behaves as the scalar field with canonical stress tensor at a flat boundary.

The rise of interest in quantum field theory in curved spacetime in the 1970's attracted renewed attention to the Casimir effect, as a more tractable model of field-theoretical effects associated with the geometry of space. It was in this new era that calculations of the energy-momentum tensor, not just the total energy, were made, and the question of the geometric covariance of the cutoff procedure was raised.

Among those who approached the subject from a general-relativistic motivation were Deutsch and Candelas [98] (1979). They consider boundary conditions (*not* quantization effects) for general curved surfaces, and for several types of quantum field theory. They avoided eigenfunction expansions by working directly with the Green functions of the elliptic operators involved.

For $x \in \Omega$ and close to the boundary of Ω, let ξ be the point on the boundary closest to x and ϵ be the distance from x to ξ. Then (ξ, ϵ) provides a convenient coordinate system in the vicinity of the smooth boundary. Deutch and Candelas [98] (1979) *assume* that near the boundary the renormalized stress tensor has an expansion of the form

$$T_{\mu\nu}(x) \sim \Sigma_{n=-4}^{\infty} A_{\mu\nu}^{(n)}(\xi)\epsilon^n. \tag{3.3}$$

Note that this is consistent with Fulling's [134] (1989) findings for the flat plate, where $\epsilon = z$. Deutch and Candelas [98] also *assume* that $A^{(n)}(\xi)$ depends only on the geometry of the boundary at ξ; that is, it can be expressed in terms of a function defining the surface and its derivatives, evaluated at ξ only. Geometrical covariance and dimensional analysis then imply that each $A^{(n)}$ is a linear combination of finitely many scalar quantities built out of the second fundamental form of the surface at ξ.

Deutch and Candelas [98] (1979) were then able to calculate the coefficients in their general series by matching it against various special cases for which the answers were known or could be easily found. For $\Omega \subset \mathcal{R}^3$ they find a hierarchy of terms. Higher-order terms are of the order ϵ^0, hence they are nonsingular. The conclusion of Deutch and Candelas [98] and [134] is that the finiteness of the Casimir force for the slab and the sphere is an accident of there geometries. In general, the electromagnetic force on a perfect conductor will turn out to be infinite. A spherical shell is unstable against wrinkling. The perfect-conductor boundary condition must be judged to be a pathological idealization in this context. Good physics requires that the infinite terms be replaced by *cutoff-dependent* terms related to the detailed properties of realistic materials.

Rosu [301] (1999) notes that if stationary, the spectrum of vacuum field noise (VFN) is an important ingredient to get information about the curvature invariants of classical worldlines (relativistic classical trajectories). For scalar quantum field vacua there are six stationary cases as shown by Letaw some time ago, these are reviewed here. However, the non-stationary vacuum noises are not out of reach and can be processed by a few mathematical methods which he briefly comments on. Since the information about the kinematical curvature invariants of the worldlines is of radiometric origin, hints are given on a more useful application to radiation and beam radiometric standards at relativistic energies.

3.3 Zero-Point Energy and Statistical Mechanics.

This section largely follow Sciama [310] (1991), a different approach can be found in Lima and Maia [391] (1995). Usually the boundary conditions associated with a physical system limit the range of normal modes that contribute to the ground state of the system and so to the zero-point energy. An example is the harmonic oscillator, which has a single normal mode of frequency ν and so has a zero-point energy of $\frac{1}{2}h\nu$. In more complicated cases the range of normal modes may depend on the configuration of the system. This would lead to a dependence of the ground-state energy on the variables defining the configuration and so, by the principle of virtual work, to the presence of an associated set of forces. One important example of such a force is the homopolar binding between two hydrogen atoms when their electric spins are antiparallel, see Hellman [170] (1927). When the protons are close together, each electron can occupy the volume around either proton. The resulting increase in the uncertainty of the electron's position leads to a decrease in the uncertainty of its momentum and so to a *decrease* in its zero-point energy. Thus, there is a binding energy associated with this diatomic configuration, and the resulting attractive force is responsible for the formation of the hydrogen molecule. By contrast, when the electron spins are parallel, the Pauli exclusion principle operates to limit the volume accessible to each electron. In this case the effective force is repulsive.

Zero-point fluctuations occur when the associated density of states is large, for example in a continuous field. In such a case the zero-point fluctuations can be an important source of *noise* and *damping*, these two phenomena being related by a fluctuation-dissipation theorem. Some examples of these zero-point effects are: *firstly* X-ray Scattering by Solids. The noise associated with zero-point fluctuations was discovered in 1914 by Debye during his study of X-ray scattering by solids. His main concern was to calculate the influence of the thermal vibrations of the lattice on the X-ray scattering, but he showed in addition that, if one assumes with Planck that the harmonic oscillators representing these vibrations have a zero-point motion, then there would be an additional scattering which would persist at the absolute zero of temperature. This additional scattering is associated with the emission of phonons by X-rays, this emission being induced by the zero-point fluctuations. Thus, noise and damping are related together as in Einstein's theory of Brownian motion. Here the example of a 'spontaneous' radiation process, which can be regarded as being induced by the coupling of the radiating system to a field of zero-point fluctuations. *Secondly* Einstein fluctuations in black body radiation. Sciama [310] (1991) notes in passing here that the interference between the zero-point and thermal fluctuations of the electromagnetic field gives a characteristic contribution to the Einstein fluctuations of the energy in a black body radiation field, namely the term 'linear' in the energy density. This term would be absent for a classical radiation field pictured as an assembly of waves. *Thirdly* the Lamb Shift. An electron, whether bound or free, is always subject to the stochastic forces produced by the zero-point fluctuations of the electromagnetic field, and as a result executes Brownian motion. The kinetic energy associated with this motion is infinite, because of the infinite energy in the high-frequency components of the zero-point fluctuations. This infinity in the kinetic energy can be removed by renormalizing the mass of the electron Weisskopf [354] (1949). As with the Casimir effect, physical significance can be given to this process in situations where one is dealing with different states of the system for which the difference in the total renormalized Brownian energy (kinetic plus potential) is finite. An example of this situation is the Lamb shift between the energies of the s and p electrons in the hydrogen atom; according to Dirac theory, the energy levels should degenerate. Welton [355] (1948) pointed out that a large part of this shift can be attributed to the effects of the induced Brownian motion of the electron, which alters the mean Coulomb potential energy. This change in electron energy is itself different for an s and p electron, and so the Dirac degeneracy is split. This theoretical effect has been well verified by the observations. One can also regard the Lamb shift as the change in zero-point energy arising from the dielectric effect of introducing a dilute distribution of hydrogen atoms into the vacuum. The frequency of each mode is simply modified by a refractive index factor, see Feynman [123] (1961), Power [279] (1966), and Barton [27] (1970).

The vacuum can be considered to be just a dissipative system. A physical system containing a large number of closely spaced modes behaves as a dissipater of energy, as well as possessing fluctuations associated with the presence of those modes. Now the vacuum states of the electromagnetic field constitute an example of a system with closely spaced energy levels, there being $(8\pi/c^3)\nu^2 \mathrm{d}\nu$ modes with frequencies lying between ν and $\nu + \mathrm{d}\nu$. Sciama [310] (1991) expects this state also to possess a dissipative character related to the zero-point fluctuations; Callen and Welton [63] (1951) proved this in their quantum mechanical derivation of the fluctuation-dissipation theorem. In this derivation the dissipation is represented simply by the *absorption* of energy by the dissipative system. This can be justified by the expectation that the energy absorbed is divided up among so many modes that the possibility that it is later re-admitted into its original modes with its initial phase relations intact can be neglected. The absorption rate is calculated by second-order perturbation theory and is found to depend quadratically on the external force acting on the system. This enables an impedance function to be defined in the usual way. This function also appears in a linear relation between the external force acting on the system and the response of the system to this force. A familiar example would be the relation between an impressed electric field, the resulting current flow, and the electrical resistance of the system. The rate of energy absorption, and hence the resistance, clearly depends on the coupling between the external disturbance and the system, and on the density of the states of the system.

Blasone *et al* [42] (2000) note the proposal that information loss in certain Casimir systems might lead to an apparent quantization of the orbits which resemble the quantum structure seen in the real world. They show that the dissipation term in the Hamiltonian for a couple of classically damped-amplified oscillators manifests itself as a geometric phase and is actually responsible for the appearance of zero-point energy in the quantum spectrum of the $1D$ linear harmonic oscillator. They also discuss the thermodynamical features of their system.

Callen and Welton [63] calculate the quantum fluctuations of a dissipative system in its unperturbed state. These fluctuations also depend on the density of states. Apparently the procedure is quite general, but for simplicity they restrict themselves to the case where the system is in thermal equilibrium at temperature T, so that its states are occupied in accordance with the Boltzmann distribution $e^{-h\nu/kT}$. By eliminating the density of states, the following relation is obtained between the mean square force $< V^2 >$ associated with the fluctuations and the frequency-dependent impedance function, $R(\nu)$:

$$< V^2 >= \frac{2}{\pi} \int_0^\infty R(\nu) E(\nu, T) \mathrm{d}\nu, \tag{3.4}$$

where

$$E(\nu, T) = \frac{1}{2} h\nu + \frac{h\nu}{e^{h\nu/kT} - 1}, \tag{3.5}$$

which is the mean energy of a harmonic oscillator at temperature T. The presence of the zero-point $\frac{1}{2}h\nu$ shows that the zero-point fluctuations (as well as the thermal fluctuations) are a source of noise power and damping, and therefore satisfy the fluctuation-dissipation theorem.

This theorem can be interpreted in three ways.

1. It shows that the damping rate R is determined by the equilibrium fluctuations $< V^2 >$, Nyquist [260] (1928) relation.

2. It shows that the noise *power* $< V^2 >$ is determined by the absorption coefficient R (Kirchhoff relation).

3. It shows how the damping rate is proportional to the density of states in the dissipative system. In fact, the mean square force $< V^2 >$ is proportional to the mean square electric field, which in turn is proportional to the energy of the dissipative system. This energy is given by the Planck distribution plus the zero-point contribution. This is the content of the fluctuation-dissipation theorem when $R(\nu)$ is regarded as proportional to the density of states, that is proportional to $\nu^2 \mathrm{d}\nu$. When the dissipative system is the vacuum ($T = 0$), this remains true. Thus, the impedance of the vacuum is proportional to ν^2.

An important illustration of these ideas was also given by Callen and Welton [63] (1951), who showed that the RADIATION DAMPING of an accelerated charge could be interpreted in this way. The non-relativistic form of this damping force is

$$\frac{2}{3}\frac{e^2}{c^3}\frac{\mathrm{d}^2 v}{\mathrm{d}t^2}, \tag{3.6}$$

and if the charge is oscillating sinusoidally with frequency ν, then the associated impedance function turns out to be

$$\frac{2}{3}\frac{e^2}{c^3}\nu^2. \tag{3.7}$$

This is precisely of the expected form, that is, proportional to ν^2, the impedance of the vacuum. Thus, the dependence of the damping force on $\mathrm{d}^2 v/\mathrm{d}t^2$ is simply a manifestation of the density of states for the vacuum electromagnetic field.

Another example is Weber's [351] (1954) discussion of the ZERO-POINT NOISE OF AN ELECTRIC CIRCUIT. One can quantize such a circuit and show that its zero-point fluctuations represent a measurable source of noise. For example, a beam of electrons passing near such a circuit would develop a noise component from this source. Associated with this noise is a damping of the electron beam caused by 'spontaneous' emission by the beam into the circuit. The zero-point noise is thus physically real, and is unaffected by a change in the origin of the energy which might be introduced to remove in a formal way the infinite total energy associated with the zero-point fluctuations of arbitrary high frequency.

The work of Callen and Welton [63] (1951) and of Weber [351] (1954) was generalized by Senitzky [315] (1960), who studied the DAMPING OF A QUANTUM HARMONIC OSCILLATOR coupled to a loss mechanism (reservoir) idealized as a system whose Hamiltonian is unspecified but which possesses a large number of closely spaced energy levels. Senitzky assumed that the coupling was switched on at a time t_0, so that for $t < t_0$ the harmonic oscillator executed its zero-point motion undisturbed by the reservoir. After t_0 the fluctuating forces exerted by the reservoir would damp out the zero-point motion of the oscillator by a now familiar mechanism. To avoid conflict with the Heisenberg uncertainty principle, he required that the zero-point fluctuations of the reservoir also introduce sufficient noise into the oscillator to restore its zero-point motion to the full quantum mechanical value $\frac{1}{2}h\nu$. Senitzky (1960) [315] showed that after a few damping times the zero-point motion of the oscillator would be effectively driven by the zero-point motions of the reservoir. This is a quantum example of the relation between noise, damping, and equilibrium which Einstein discovered in 1905.

RESPONSE THEORY was initiated by Onsager [262] (1931), who extended Einstein's theory of Brownian motion by taking into account the perturbation of the dissipative system by the system being dissipated. He related this perturbation to the equilibrium fluctuations of the unperturbed dissipative system by an ansatz which was very much in Einstein's spirit. He assumed that if, as a result of these fluctuations, the system at one instant deviated appreciably from its mean configuration, then on average it would regress back to the mean at the same rate as if the deviation had been produced by an external perturbation.

This average regression would represent an irreversible approach to equilibrium and so would determine the generalized friction coefficient associated with the response of the system to the external perturbation. In the approximation he envisaged, the perturbation would have a linear effect, and as an example consider an electric current as the response to an impressed voltage. The linear coefficient of this response, the resistance, not only would govern the rate at which the current would die irreversibly away after the voltage is removed, but also would govern the rate of dissipation associated with Joule heating. With Onsager's ansatz it is expected that this resistance is determined by the equilibrium fluctuations of the unperturbed system, giving a further reason for obtaining a fluctuation-dissipation relation.

Of course, the friction coefficient determines only the out-of-phase response of the system. However, one would also expect the in-phase response, that is the reactance of the system, to be determined by the equilibrium fluctuation spectrum. This follows from the dispersion relations which are a direct consequence of the causality requirement that the response of the system should not precede the disturbance of it. The reactance at any frequency can then be determined by an integral of the friction overall frequencies; the friction is, of course, itself determined by the fluctuation spectrum.

One can also calculate the total response directly: this was first done by Kubo [210] (1957). Kubo used the fact that an averaging of an observable over a system in thermal equilibrium involves multiplying the observable by $E^{-H/kT}$, where H is the Hamiltonian of the system and T is the temperature. This is very similar to multiplying the observable by a quantum mechanical time-evolution operator, with the temperature acting as the reciprocal of an imaginary time. By exploiting this analogy, and using complex variable methods, Kubo arrived at the following fluctuation-response relation:

$$\sigma_{ab}(\nu) = P(\nu, T) \int_{-\infty}^{\infty} e^{-i\nu t} < j_a(t) j_b(0) > \mathrm{d}t, \tag{3.8}$$

where the complex conductivity σ_{ab} is given in terms of an external field $E_b(\nu)$ by

$$j_a = \sigma_{ab} B_b, \tag{3.9}$$

$P(\nu, T)$ is the Plankian function (with zero-point contribution),

$$P(\nu, T) = \frac{1 - e^{-h\nu/kT}}{2\nu}, \tag{3.10}$$

and $< j_a(t) j_b(0) >=< j_a(0) j_b(t) >$ represents the quantum thermal correlation function of the unperturbed current fluctuations in the system. Comparing with equation 1.2

$$2B(\nu, T) P(\nu, T) = h \exp -\frac{h\nu}{kT}. \tag{3.11}$$

Note, in particular, that by microscopic reversibility,

$$< j_a(t) j_b(0) =< j_a(0) j_b(t) >, \tag{3.12}$$

so that

$$\sigma_{ab} = \sigma_{ba} \quad ; \tag{3.13}$$

these are just Onsager's [262] (1931) reciprocal relations.

Some recent work on statistical mechanics and the Casimir effect includes Power and Thirunamachandran [280] (1994) who find in a systematic way the fully retarded dispersion interaction potentials, including many-body interactions, among neutral molecules. The method used relates the total zero-point energy of all the electromagnetic modes with the spectral sum of a linear operator. The difference between the zero-point energies with and without molecules present is given as a contour integral. From the value of this integral it is possible to extract the N-body dispersion energy by locating those terms which depend on the product of the polarizabilities of those N molecules. The Casimir-Polder pairwise energy is the two-body result. General formulas are found, and the special cases for N=3 and 4 are discussed in detail. The non-retarded interaction potentials are found as their asymptotic limits for small intermolecular separations, and the London and Axilrod-Teller results are the N=2 and N=3 special cases. The N=4 near zero limit is presented in its explicit form. It is of interest to note that, for the one-body case, the energy shift given by this method is the nonrelativistic Lamb shift.

Herzog and Bergou [173] (1997) investigate specific nonclassical maximum-entropy states of a simple harmonic-oscillator mode that arise when only number states (Fock states) differing by a multiple of a certain integer $k(k \geq 1)$ are allowed to be occupied. For $k = 2$ the number-probability distribution of the even-number maximum-entropy state has a close resemblance to that of a squeezed vacuum state. These maximum entropy states can be obtained as the stationary solutions of a master equation which takes into account k-quantum absorption as well as k-quantum emission processes only. The steady-state solution of this master equation depends on their initial conditions. For the vibrational motion of a trapped ion such nonclassical maximum-entropy states could be produced with the help of the recently proposed method of laser-assisted quantum reservoir engineering.

Rosu [301] (2000) discusses vacuum field noise, see the end of §3.2 above.

Some recent work on the Casimir effect and the liquid-crystal ground state includes Kivelson *et al* [199] (1998) who notes that the character of the ground state of an antiferromagnetic insulator is fundamentally altered following addition of even a small amount of charge. The added charge is concentrated into domain walls across which a π phase shift in the spin correlations of the host material is induced. In two dimensions, these domain walls are 'stripes' which can be insulating or conducting - that is, metallic 'rivers' with their own low -energy degrees of freedom. However, in arrays of one-dimensional metals, which occur in materials such as organic conductors, interactions between strings typically drive a transition to an insulating ordered charge-density-wave (CDW) state at low temperatures. Here they show that such a transition is eliminated if the zero-point energy of transverse stripe fluctuations is sufficiently large compared to the CDW coupling stripes. As a consequence, there should exist electronic quantum liquid-crystal phases, which constitute new states of matter, and which can be either high-temperature superconductors or two-dimensional anisotropic 'metallic' non-Fermi liquids. Neutron scattering and other experiments in the copper oxide superconductors $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$ already provide evidence for the existence of these phases in at least one class of materials.

Sentizky [316] (1998) studies the effects of zero-point energy, in comparison with those of excitation energy, are investigated by studying the behaviour of two coupled harmonic oscillators for four types of coupling: rotating-wave coupling, counter-rotating wave coupling, dipole-dipole (or dipole field) coupling and elastic-attraction coupling. It is found that in all but the rotating-wave coupling case, which has only slowly varying terms in their interaction Hamiltonian, the interaction coupling between the ground-state oscillators is followed by an oscillating energy increase in both systems has a lower zero-point energy that the energy of the state in which the individual oscillators are in their ground states, thus leading to the presence of a van der Waals attraction. This result motivates the conclusion that only the rapidly varying terms in the interaction Hamiltonian are responsible for the attraction. The energies of the individual oscillators while the coupled system is in its ground state are calculated. In the van der Waals cases, these energies tern out to be greater than the zero-point energies of the individual oscillators. he offers a physical explanation for the increase. he discusses the feasibility of the utilization or absorption of zero-point energy.

3.4 Casimir Calculations.

There are a very large number of papers calculating the Casimir effect, some recent ones are mentioned below. The technique of *zeta*-function regularization which is used for many of these calculations is discussed in the book of Elizade *et al* (1994) [378].

Duru and Tomak [108] (1993) discuss the magnitude of the Casimir energies for the electrons and the electromagnetic fields resulting from confinement of the corresponding particles in dis-ordered materials. They estimate that the vacuum energy of the confined light is of the order of eV, while for electrons it is negligible small.

Fendley *et al* [121] (1993) study the spectrum, the massless S-matrices and the ground-state energy of the flows between successive minimal models of conformal field theory, and within the sine-Gordon model with imaginary coefficient of the cosine term (related to the minimal models by "truncation"). For the minimal models, they find exact S-matrices which describe the scattering of massless kinks, and show using the thermodynamic Bethe ansatz that the resulting non-perturbative c-function (defined by the Casimir energy on a cylinder) flows appropriately between the two theories, as conjectured earlier. For the non-unitary sine-Gordon model, they find unusual behavior. For the range of couplings they can study analytically, the natural S-matrix deduced from the minimal one by "undoing" the quantum-group truncation does not reproduce the proper c-function with the TBA. They say that it does describe the correct properties of the model in a magnetic field.

Vanzo [343] (1993) discusses the functional integral for a scalar field confined in a cavity and subjected to linear boundary conditions. He shows how the functional measure can be conveniently dealt with by modifying the classical action with boundary corrections. The nonuniqueness of the boundary actions is described with a three-parameter family of them giving initial boundary conditions. In some cases, the corresponding Greens' function will define a kind of generalized Gaussian measure on function space. He discusses vacuum energy, paying due attention to its anomalous scale dependence, and the physical issues involved are considered.

Bender and Kimball [37] (1994) compute the Casimir force on a D-dimensional sphere due to the confinement of a massless scalar field as a function of D, where D is a continuous variable that ranges from $-\infty$ to ∞. The dependence of the force on the dimension is obtained using a simple and straightforward Green's function technique. They find that the Casimir force vanishes as $D \to +\infty$ (D non-even integer) and also vanishes when D is a negative even integer. The force has simple poles at positive even integer values of D.

Villarreal [347] (1995) investigates the effective potential up to the two-loop level for a scalar field with a quartic self-interaction defined on a Minkowskian spacetime endowed with a nontrivial topology $T^4 = S^1 \times S^1 \times S^1 \times S^1$. The periodicity in the time dimension allows one to get simultaneously expressions for the effective potential and the self-energy valid for arbitrary temperature. The spatial periodicity generates Casimir forces that compactify the associated spatial dimensions. He finds that for a critical value of the coupling constant the self-interaction of the field balances the Casimir interaction, and a "phase transition" occurs, creating a stable vacuum structure with the topology of a two-torus. He says that thermal fluctuations might destroy this structure in another phase transition occurring at high temperature. Closely related to the work of Villarreal [347] is the work Elizalde and Kirsten [379] (1994) on topological symmetry breaking in self-interacting theories on toroidal spacetime.

Garattini [143] (1999) calculates by variational methods, the Schwarzschild and flat space energy difference following the subtraction procedure for manifolds with boundaries, calculate by variational methods, the Schwarzschild and flat space energy difference. He considers the one loop approximation for TT tensors. An analogy between the computed energy difference in momentum space and the Casimir effect is illustrated. He finds a singular behaviour in the ultraviolet-limit, due to the presence of the horizon when $r = 2m$. He finds $r > 2m$ this singular behaviour disappears, which is in agreement with various other models previously presented.

Leseduarte and Romeo [221] (1996) study the simultaneous influence of boundary conditions and external fields on quantum fluctuations by considering vacuum zero-point energies for quantum fields in the presence of a magnetic fluxon confined by a bag, circular and spherical for bosons and circular for fermions. They calculate Casimir effect in a generalized cut-off regularization after applying ζ-function techniques to eigenmode sums and using recent techniques involving Bessel ζ-functions at negative arguments.

Krech and Landau [207] (1996) note that if a critical system is confined to a finite geometry critical fluctuations of the order parameter generate long-range forces between the system boundaries. These Casimir forces, are characterized by universal amplitudes and scaling functions. They derive a hybrid Monte Carlo algorithm and used it to measure the Casimir amplitudes directly and accurately. They apply the algorithm to a critical q-state Potts model confined to a rectangular MXL geometry in d=2 dimensions and to a critical Ising model confined to a $M^2 \times L$ geometry in d=3 dimensions. They find good agreement with rigorous results in d=2 and compare their results with field-theoretical estimates of the Casimir amplitude in d=3.

Weingert [352] (1996) discusses free quantum electromagnetic radiation is enclosed in a one dimensional cavity. The contribution of the kth mode of the field to the energy, contain in a region $\mathcal{R}$ of the cavity, is minimized. For the resulting *squeezed* state, the energy expect ion in $\mathcal{R}$ is *below* its vacuum value. Pressure zero-point energy out of the spatial region can be used to temporarily *increase* the Casimir force.

Bayin and Mustafa [31] (1997) calculate the Casimir energy of the massless conformal scalar field on the surface (S-2) of a 3-dimensional Reimann sphere by using the point-splitting, mode sum and the ζ-function renormalization methods. They also consider the half space case with both Dirichlet and the Neuwman boundary conditions. This problem is interesting since the Casimir energy could be calculated analytically by various methods, thus allowing them to compare different regularization schemes.

Williams [357] (1997) discusses how topology and Casimir might be related, see also §2.14

Bukina and Shtykov [59] (1997) evaluate the effective one-loop potential on $R^{m+1} \times S^N$ spaces for massless tensor fields is evaluated. The Casimir energy is given as a value of $\zeta-$ function by means of which regularization is made. In even-dimensional spaces the vacuum energy contains divergent terms coming from poles of $\zeta(s, q)$ at $s = 1$, whereas in odd-dimensional spaces it becomes finite.

Gosdzinsky and Romeo [151] (1998) find values for the vacuum energy of scalar fields under Dirichlet and Neuman boundary conditions on an infinite cylindrical surface are found, and they happen to be of opposite signs. In contrast with classical works, they apply a complete ζ-function regularization scheme. These fields are regarded as interesting both by themselves and as the key to describing the electromagnetic case. They claim that the electromagnetic Casimir effect in the presence of this surface, found by De Raad and Milton, is now confirmed.

Lu and Huang (1998) [231] find that "unlike a closed string the Casmir energy of an open string can be either positive or negative".

Cognola *et al* [83] (1999) calculate Casimir energies, in an arbitrary number of dimensions, for massless quantized fields in spherically symmetric cavities is carried out. All the most common situations, including scalar and spinor fields, the electromagnetic field, and various boundary conditions are treated with care. The final results are given as analytical closed expressions in terms of Barnes

ζ-functions. They perform a direct, straightforward numerical evaluation of the formulas is then performed, which yields highly accurate numbers of, in principle, arbitrarily good precision.

Katzgraber *et al* [193] (1999) calculate the attractive long-range vortex-vortex interactions of the van der Waals type presented in anisotropic and layered superconductors. This allows them to define a 2D Casimir problem and determine the attractive force between two half planes.

Kirsten [388] (1999) among other things discusses the use of spectral functions in calculation of Casimir energy. He says thatseveral functions of the spectrum of second order elliptic differential operators play a central role in the analysis of properties of physical systems. E.g. in statistical mechanics relevant spectral functions comprise of various partition sums for the evaluation of thermodynamical quantities as critical temperatures or fluctuations of the ground state occupation. In quantum field theory under external conditions relevant quantities are effective actions (closely related to functional determinants) and ground state or vacuum energies, which describe e.g. the influence of external fields or of boundaries on the properties of the vacuum. In this context, results are generically divergent and need a renormalization to give a physical meaning to them. The renormalization procedure at one-loop is completely determined by the heat-kernel coefficients, central objects of spectral geometry. All mentioned spectral functions can be related to an associated ζ-function. In recent years an analysis of ζ-functions in spherically symmetric situations has become available. These results (and techniques involved) allow the analysis of vacuum properties in the presence of spherically symmetric boundaries or background fields as well as the determination of thermodynamical properties of ideal gases in magnetic traps. Furthermore, when combined with other methods, it provides an effective scheme for the calculation of heat-kernel coefficients on arbitrary smooth Riemannian manifolds with smooth boundaries.

Lambiase *et al* [212] (1999) propose a simple method is proposed to construct the spectral ζ-functions required for calculating the electromagnetic vacuum energy with boundary conditions given on a sphere or on an infinite cylinder. When calculating the Casimir energy in this approach no exact divergences appear and no renormalization is needed. The starting point of the consideration is the representation of the ζ-functions in terms of contour integral, further the uniform asymptotic expansion of the Bessel function is essentially used by them. After the analytic continuation, needed for calculating the Casimir energy, the ζ-functions are presented as infinite series containing the Riemann ζ-function with rapidly falling down terms. The spectral ζ-functions are constructed exactly for a material ball and infinite cylinder placed in an uniform endless medium under the condition that the velocity of light does not change when crossing the interface. As a special case, perfectly conducting spherical and cylindrical shells are also considered in the same line. In this approach they claim to succeed in justifying in mathematically rigorous way, the appearance of the contribution to the Casimir energy for cylinder which is proportional to $\ln(2\pi)$. This method is related to that of Leseduarte and Romeo [389] (1996) and Bordag *et al* [371] (1996).

Matloob [242] (1999) uses the Maxwell stress tensor to introduce the radiation pressure force of the electron field on a conducting surface. This expression is related to the imaginary part of the vector potential Green function for the fluctuating fields of the vacuum via the fluctuation dissipation theorem and Kubo's formula see §3.2. The formalism allows him to evaluate the vacuum radiation pressure on a conducting surface without resorting to the process of field quantization. The later formula is used to calculate the attractive and repulsive Casimir force between two conducting plates. What is more, in this formalism, there is no need to apply any regularization proceedure to recover the final result.

Bordag and Kirsten [372] (1996) and Bordag *et al* [373] (2000) consider the vacuum energy of a scalar field in a spherically symmetric background field. The numerical procedure is refined from previous work and applied to several examples. They provide numerical evidence that repulsive potentials lead to positive contributions to the vacuum energy, and show the crucial role played by bound-states.

Bordag and Vassilevich [48] (2000) calculate the Casimir force between two parallel plates if the boundary conditions for the photons are modified due to presence of the Chern-Simons term. They show that this effect should be measurable within present experimental technique.

Brevik *et al* [54] (2000) apply the background field method and the effective action formalism to describe the four-dimensional dynamical Casimir effect. Their picture corresponds to the consideration of quantum cosmology for an expanding FRW universe (the boundary conditions act as a moving mirror) filled by a quantum massless GUT which is conformally invariant. They consider cases in which the static Casimir energy is repulsive and attractive. Inserting the simplest possible inertial term, they find, in the adiabatic (and semiclassical) approximation, the dynamical evolution of the scale factor and the dynamical Casimir stress analytically and numerically (for SU(2) super Yang-Mills theory). Alternative kinetic energy terms are also explored.

Garattini [380] (2000) calculates by variational methods the subtraction procedure for manifolds with boundaries the Köttler (Schwarzschild-deSitter) and the deSitter space energy difference. By computing the one loop approximation for TT tensors he discovers the existence of an unstable mode even for the non-degenerate case. This result seems to be in agreement with the sub-maximal black hole pair creation of Bousso-Hawking. The instability can be eliminated by the boundary reduction method. He discusses the implications for a foam-like space.

Herdegen [171] (2000) notes that two thin conducting, electrically neutral, parallel plates forming an isolated system in vacuum exert attracting force on each other, whose origin is the quantum electrodynamical interaction. He says that this theoretical hypothesis, known as Casimir effect, has been also confirmed experimentally. Despite long history of the subject, no completely convincing theoretical analysis of this effect appears in the literature. Here he discusses the effect (for the scalar field) anew, on a revised physical and mathematical basis. He uses standard, but advanced methods of relativistic quantum theory. No anomalous features of the conventional approaches appear. The Casimir quantitative prediction for the force is shown to constitute the leading asymptotic term, for large separation of the plates, of the full, model-dependent expression.

Marachevsky [240] (2000) applies a general formalism of quantum field theory and addition theorem for Bessel functions to derive formula for Casimir-Polder energy of interaction between a polarizable particle and a dilute dielectric ball. He shows The equivalence of dipole-dipole interaction and Casimir energy for dilute homogeneous dielectrics. He uses a novel method to derive Casimir energy of a dilute dielectric ball without divergences in calculations. Physically realistic model of a dilute ball is discussed. He reviews different approaches to the calculation of Casimir energy of a dielectric ball.

Scandurra [305] (2000) computes the ground state energy of a massive scalar field in the background of a cylindrical shell whose potential is given by a δ-function. The zero-point energy is expressed in terms of the Jost function of the related scattering problem, the renormalization is performed with the help of the heat kernel expansion. The energy is found to be negative for attractive and for repulsive backgrounds as well.

Hagen [403] (2001) considers the Casimir force between two conducting planes in both the electromagnetic and scalar field cases. This is done by the usual summation over energy eigenmodes of the system as well as by a calculation of the stress tensor in the region between the planes. The latter case requires that careful attention be given to singular operator products, an issue which is accommodated here by invoking the point separation method in conjunction with a scalar cutoff. This is shown to yield cutoff dependent and divergent contributions to the Casimir pressure which are dependent on the separation parameters, but entirely consistent with Lorentz covariance. Averaging over the point splitting parameters allows finite results to be obtained, but fails to yield a unique Casimir force.

3.5 The Dynamical Casimir Effect.

In the past, the Casimir effect has been considered as a *static* effect. Growing interest in recent years has been drawn to the *dynamical* variety of the effect, meaning, in essence, that not only the geometric configurations of the extremal boundaries (such as plates) but also their velocities play a physical role, see Brevik *et al* [54] (2000) and §4.7. One can also consider the Universe to form a sort of boundary, in which case the "Casimir Effect" is just QFT in some cosmological model, see §3.9. Padmanabhan and Coudhury [265] (2000) use the fact that the Universe forms some sort of boundary to move from some sort of quantum gravity theory to QFT in curved spaces. Having the Universe forming some sort of boundary can be thought of as being an aspect of Mach's principle, compare Roberts [296] (1998). There is a method is based on the requirement that the zero-point fluctuations must vanish on the boundary. This requirement leads to a finite change in the zero-point energy, which can be extracted from the infinite total energy by special techniques. The two-dimensional problem can be solved exactly by de Witt's point splitting method discussed in de Witt [99] (1975) and Fulling and Davies [135] (1975), and one again finds a radiation damping force which in the non-relativistic limit is proportional to $\mathrm{d}^2v/\mathrm{d}t^2$. Some recent dynamical Casimir calculations are mentioned below.

Langfeld *et al* [216] (1993), compare §2.11 study non-trivial ϕ^4-theory is studied in a renormalisation group invariant approach inside a box consisting of rectangular plates and where the scalar modes satisfy periodic boundary conditions at the plates. They find that the Casimir energy exponentially approaches the infinite volume limit, the decay rate given by the scalar condensate. It therefore essentially differs from the power law of a free theory. This might provide experimental access to properties of the non-trivial vacuum. At small interplate distances the system can no longer tolerate a scalar condensate, and a first order phase transition to the perturbative phase occurs. The dependence of the vacuum energy density and the scalar condensate on the box dimensions are presented.

Pontual and Moraes [278] (1996) show that the existence of a non-zero vacuum energy density (the Casimir energy) for a scalar field appears in a continuous elastic solid due to the presence of a topological defect, the screw dislocation. An exact expression is obtained for this energy density in terms of the Burgers vector describing the defect, for zero and finite temperature.

Carlson *et al* [66] (1997) note that in a recent series of papers, Schwinger discussed a process that he calls the dynamical Casimir effect. The key essence of this effect is the change in zero-point energy associated with any change in a dielectric medium. In particular, if the change in the dielectric medium is taken to be the growth or collapse of a bubble, this effect may have relevance to sonoluminescence, see §4.7.The kernel of Schwinger's result is that the change in Casimir energy is proportional to the change in volume of the dielectric, plus finite-volume corrections. They say that other papers have called into question this result, claiming that the volume term should actually be discarded, and that the dominant term remaining is proportional to the surface area of the dielectric. They claim to present a careful and critical review of the relevant analyses. They find that the Casimir energy, defined as the change in zero-point energy due to a change in the medium, has at leading order a bulk volume dependence. This is in full agreement with Schwinger's result, once the correct physical question is asked. They say they have nothing new to say about sonoluminescence itself.

Li *et al* [223] (1997) re-examine the Casimir effect giving rise to an attractive or repulsive force between the configuration boundaries that confine the massless scalar field for a $(D-1)$-dimensional rectangular cavity with unequal finite p edges and different spacetime dimensions D. That say that with periodic or Neumann boundary conditions, the energy is always negative. The case of Dirichlet boundary conditions is more complicated. The sign of the Casimir energy satisfying Dirichlet conditions on the surface of a hypercube (a cavity with equal finite p edges) depends on whether p is even or odd. In the general case (a cavity with unequal p edges), however, we show that the sign of the Casimir energy does not only depend on whether p is odd or even. Furthermore, they find that the Casimir force is always attractive if the edges are chosen appropriately. It is interesting that the Casimir force may be repulsive for odd p cavity with unequal edges, in contrast with the same problem in a hypercube case.

Sitenko and Babansky [319] (1997) study the combined effect of the magnetic field background in the form of a singular vortex and the Dirichlet boundary condition at the location of the vortex on the vacuum of quantized scalar field. They find the induced vacuum energy density and current to be periodic functions of the vortex flux and holomorphic functions of the space dimension.

Dalvit and Neto [90] (2000) derive a master equation for a mirror interacting with the vacuum field via radiation pressure. They say that the dynamical Casimir effect leads to decoherence of a 'Schrödinger cat' state in a time scale that depends on the degree of 'macroscopicity' of the state components, and which may be much shorter than the relaxation time scale. Furthermore they say that coherent states are selected by the interaction as pointer states.

Hofmann *et al* (2000) study the stabilization of one spatial dimension in $p+1+1$-dimensional spacetime in the presence of p-dimensional brane(s), a bulk cosmological constant and the Casimir force generated by a conformally coupled scalar field. They find general static solutions to the metric which require two fine tunings: setting the size of the extra dimension to its extremal value and the effective cosmological constant to zero. Taking these solutions as a background configuration, they perform a dimensional reduction and study the effective theory in the case of one- and two-brane configurations. They show that the radion field can have a positive mass squared, which corresponds to a stabilization of the extra dimension, only for a repulsive nature of the Casimir force. This type of solution requires the presence of a negative tension brane. The solutions with one or two positive tension branes arising in this theory turn out to have negative radion mass squared, and therefore are not stable.

Milton [246] (2000) notes that zero-point fluctuations in quantum fields give rise to observable forces between material bodies, which he notes are the so-called "Casimir forces". In this paper he presents some results of the theory of the Casimir effect, primarily formulated in terms of Green's functions. There is an intimate relation between the Casimir effect and van der Waals forces. Applications to conductors and dielectric bodies of various shapes are given for the cases of scalar, electromagnetic, and fermionic fields. The dimensional dependence of the effect is described. Finally, he asks the question: Is there a connection between the Casimir effect and the phenomenon of sonoluminescence? Sonoluminescence is discussed in §4.7.

3.6 Mechanical Analogs of the Casimir Effect.

The Casimir effect has various analogs some of which are mentioned below.

Sokolov [322] (1994) considers energy production due to the Casimir effect for the case of a superdense state of matter, which can appear in such cosmological objects as white dwarfs, neutron stars, quasars and so on. The energy output produced by the Casimir effect during the creation of a neutron star turns out to be sufficient to explain nova and supernova explosions. He claims to show that the Casimir effect might be a possible source of the huge energy output of quasars.

Van Enk [118] (1995) shows that the vacuum can induce a torque between two uniaxial birefringent dielectric plates. He discuss the differences and analogies between this torque and the Casimir force in the same configuration. He show that the torque can be interpreted as arising from angular-momentum transfer from the vacuum to the plates, as well as from the orientation dependence of the zero-point energy.

Boersma [45] (1996) notes that at sea, on a windless day, in a strong swell, free floating ships will roll heavily. It was believed in the days of the clipper ships that under those circumstances two vessels at close distances will attract each other, Boersma [45] asks if they do. The ships are harmonic oscillators in a wave field and as such analogous to two atoms in the sea of vacuum fluctuations. These atoms do attract by the van der Waals force, suggesting that the two vessels attract.

Larraza and Denardo [217] (1998) present theoretical and experimental results for the force law between two rigid, parallel plates due to the radiation pressure of band-limited acoustic noise. They claim excellent agreement is shown between theory and experiment. While these results constitute an acoustic analog for the Casimir effect, an important difference is that the band-limited noise can cause the force to be *attractive* or *repulsive* as a function of the distance of separation of the plates. Applications of the acoustic Casimir effect to background noise transduction and non-resonant acoustic levitation are suggested.

Widom *et al* [356] (1998) note that one loop field theory calculations of free energies quite often yield violations of the stability conditions associated with the thermodynamic second law. Perhaps the best known example involves the equation of state of black holes. They point out that the Casimir force between two parallel conducting plates also violates a thermodynamic stability condition normally associated with the second law of thermodynamics.

Mulhopaclhgay and Law [253] (1999) study the critical Casimir force per unit area which determines the film thickness of critical binary liquid wetting films for the specific case of opposite boundary conditions within the film. A universal scaling function is derived in the one-phase region. At criticality this scaling function reduces to a universal amplitude with value $\Delta_{+,-} \sim 0.0053$

3.7 Applications of the Casimir Effect.

Iacopini [179] (1993) notes that the possibility of observing the Casimir force at macroscopic distances (a few centimeters) using a conformal optical resonator is discussed and a possible experimental apparatus is also suggested. Long range effects are also discussed in Spruch [326] (1996), Yam [361] (1997) and Winterberg [359] (1998). Kiers and Tytgat [196] (1997) note that it has recently been argued that long range forces due to the exchange of massless neutrinos give rise to a very large self-energy in a dense, finite-ranged, weakly-charged medium. Such an effect, if real, would destabilize a neutron star. To address this issue they have studied the related problem of a massless neutrino field in the presence of an external, static electroweak potential of finite range. To be precise, they have computed to one loop the exact vacuum energy for the case of a spherical square well potential of depth α and radius R. For small wells, the vacuum energy is reliably determined by a perturbative expansion in the external potential. For large wells, however, the perturbative expansion breaks down. A manifestation of this breakdown is that the vacuum carries a non-zero neutrino charge. The energy and neutrino charge of the ground state are, to a good approximation for large wells, those of a neutrino condensate with chemical potential $\mu = \alpha$. Our results demonstrate explicitly that long-range forces due to the exchange of massless neutrinos do not threaten the stability of neutron stars.

3.8 Experimental Testing of the Casimir Effect.

An early experimental verification of the Casimir effect is that of Tabor and Winton [334] (1969), see also §3.3¶5 above. Some more recent measurement are mentioned below.

Storry *et al* [329] (1995) make measurements which they compare with high-precision variational calculations. They find that the expected long-range Casimir effect is not present.

Grado *et al* [152] (1999) notes that the accurate measurement of the Casimir force and the search for hypothetical long-range interactions are subjects of growing interest. They propose to use a suspended interferometric device to measure the Casimir effect between metallic flat surfaces. This allows them to perform the measurement at distances much larger than the ones used in previous measurements with spherical surfaces. The use of the proposed dynamic detection scheme can also help in the discrimination of the different contributions to the measured force. Some considerations about the physical information which can be obtained from this type of measurement are also made.

Lamoreaux [214] (1999) reviews the recent experimental verifications of the Casimir force between extended bodies. He says that with modern techniques, it now appears feasible to test the force law with 1% precision; he addresses the issues relating to the interpretation of experiments at this level of accuracy

3.9 Quantum Field Theory on Curved Spaces.

The Casimir effect as originally understood depends crucially on the two plate boundaries. One can relax this requirement to that where the boundaries are moving , and then this effect is very close to the sort of situations which are studied in QFT on curved spacetimes, where the role of the boundary is now taken over by the metric tensor. Padmanabhan and Choudhury [265] (2000), see §4.7 above, try to justify QFT on curved spacetimes as a well defined limit from so as yet unknown theory of quantum gravity. Meissner and Veneziano [244] (1991) discuss how to produce spacetime invariant vacua. There are many papers on this topic; some recent calculations, which specifically mention vacuum energy, are mentioned below.

Svaiter and Svaiter [331] (1993) discuss analytic regularization methods used to obtain the renormalized vacuum energy of quantum fields in an arbitrary ultrastatic spacetime. After proving that the ζ-function method is equivalent to the cutoff method with the subtraction of the polar terms, they present two examples where the analytic extension method gives a finite result, but in disagreement with the cutoff and ζ-function methods.

Linet [229] (1994) determines generally the spinor Green's function and the twisted spinor Green's function in an Euclidean space with a conical-type line singularity. In particular, in the neighbourhood of the point source, he expresses them as a sum of the usual Euclidean spinor Green's function and a regular term. In four dimensions, he uses these determinations to calculate the vacuum energy density and the twisted one for a massless spinor field in the spacetime of a straight cosmic string. In the Minkowski spacetime, he determines explicitly the vacuum energy density for a massive twisted spinor field.

Soleng [323] (1994) notes that quantum field theory in curved spacetime implies that the strong equivalence principle is violated outside a spherically symmetric, static star. Here he assumes that the quantum gravity effects restore the strong equivalence principle. Together with the assumption that the effective vacuum polarization energy-momentum tensor is traceless, this leads to a specific algebraic form of the energy-momentum tensor for which an exact solution of Einstein's field equations is found. The solution gives the post-Newtonian parameters $\gamma = 1$ and $\beta = 1 + 3\delta$, where δ is a dimensionless constant which determines the energy density of the anisotropic vacuum. The vacuum energy changes the perihelion precession by a factor of $1 - \delta$.

Allen *et al* [11] (1995) combine and further develop their ideas and techniques for calculating the long range effects of cosmic string cores on classical and quantum field quantities far from an (infinitely long, straight) cosmic string. They find analytical approximations for (a) the gravity-induced ground state renormalized expectation values of $\hat{\varphi}^2$ and $\hat{T}_\mu{}^\nu$ for a non-minimally coupled quantum scalar field far from a cosmic string (b) the classical electrostatic self force on a test charge far from a superconducting cosmic string. Surprisingly – even at cosmologically large distances – all these quantities would be very badly approximated by idealizing the string as having zero thickness and imposing regular boundary conditions; instead they are well approximated by suitably fitted strengths of logarithmic divergence at the string core. Their formula for $\langle\hat{\varphi}^2\rangle$ reproduces, with much less effort and much more generality, the earlier numerical results of Allen and Ottewill. Both $\langle\hat{\varphi}^2\rangle$ and $\langle\hat{T}_\mu{}^\nu\rangle$ turn out to be "weak field topological invariants" depending on the details of the string core only through the minimal coupling parameter "ξ" (and the deficit angle). Their formula for the self-force (leaving aside relatively tiny gravitational corrections) turns out to be attractive: They obtain, for the self-potential of a test charge Q a distance r from a (GUT scale) superconducting string, the formula $-Q^2/(16\epsilon_0 r \ln(qr))$ where q is an (in principle, computable) constant of the order of the inverse string radius.

Bytsenko *et al* [62] (1995) review the heat-kernel expansion and ζ-regularization techniques for quantum field theory and extended objects on curved space-times.

In particular they discuss ultrastatic space-times with spatial section consisting in manifold with constant curvature in detail. They present several mathematical results, relevant to physical applications, including exact solutions of the heat-kernel equation, a simple exposition of hyperbolic geometry and an elementary derivation of the Selberg trace formula. They consider with regards to the physical applications, the vacuum energy for scalar fields, the one-loop renormalization of a self-interacting scalar field theory on a hyperbolic space-time, with a discussion on the topological symmetry breaking, the finite temperature effects and the Bose-Einstein condensation. They also present some attempts to generalize the results to extended objects, including some remarks on path integral quantization, asymptotic properties of extended objects and a novel representation for the one-loop superstring free energy.

Davies *et al* (1996) derive conditions for rotating particle detectors to respond in a variety of bounded spacetimes and compare the results with the folklore that particle detectors do not respond in the vacuum state appropriately to their motions. The breifly address applications involoving possible violations of the second law of thermodynamics.

Chan [72] (1997) discusses vacuum energy for static backgrounds.

Ford and Svaiter [129] (1998) note that the imposition of boundary conditions upon a quantized field can lead to singular energy densities on the boundary. They treat the boundaries as quantum mechanical objects with a nonzero position uncertainty, and show that the singular energy density is removed. They say that this treatment also resolves a long standing paradox concerning the total energy of the minimally coupled and conformally coupled scalar fields.

Vilkovisky [346] (1999) presents a solution to the simplest problem about the vacuum backreaction on a pair creating source. The backreaction effect is nonanalytic in the coupling constant and restores completely the energy conservation law. The vacuum changes the kinematics of motion like relativity theory does and imposes a new upper bound on the velocity of the source.

Hu and Phillips [178] (2000) note that from calculations of the variance of fluctuations and of the mean of the energy density of a massless scalar field in the Minkowski vacuum as a function of an intrinsic scale defined by the world function between two nearby points (as used in point separation regularization) they claim that, contrary to prior claims, the ratio of variance to mean-squared being of the order unity does not imply a failure of semiclassical gravity. It is more a consequence of the quantum nature of the state of matter field than any inadequacy of the theory of spacetime with quantum matter as source.

4 Vacuum Energy on Large Scales.

4.1 The Cosmological Constant.

The standard picture of how vacuum energy gives rise to as cosmological constant is outlined in §2.1 at the end and my caveats to this are given in §2.1 end of ¶2. To reiterate *two* of the problems. *Firstly* that the cosmological constant, found by the standard method, is time independent, and a cosmological constant can be modeled by a perfect fluid with pressure and density of opposite signs, such a choice allows time dependence as pressure and density can be time dependent. *Secondly* a cosmological constant implies that either the equivalent pressure or density must be negative. I have discussed, Roberts [286] (1987), how the cosmological constant might alter the orbit of Pluto, and I have discussed, Roberts [297] (1998), how it might be possible to re-interpret the cosmological constant as the object on non-metricity. Papers on the cosmological constant are written are the rate of about one a day, here I restrict myself to about 30 recent papers.

Four recent reviews are: Carroll [67] (2000), Weinberg [353] (2000) Garriga and Vilenkin [144] (2000), Rugh and Zinkernagel [396] (2000). Focusing on recent developments, Carroll presents a pedagogical overview of cosmology in the presence of a cosmological constant, observational constraints on its magnitude, and the physics of a small (and potentially nonzero) vacuum energy, focusing on recent developments. Weinberg's approach is given in more detail in §4.2

Sciama [310] (1991) approach to the cosmological constant is as follows. If the vacuum energy density is a local quantity, one would expect that, for reasons of symmetry, it would have to have the form $\lambda g_{\nu\mu}$, and so correspond to the 'cosmological' term in Einstein's field equations. This is redolent of the related problem that, in grand unified and supergravity type theories, one would expect to have a cosmological term of order 10^{120} times greater than any observational cosmology. Some very fine-tuned cancellations seem to be required to achieve agreement with observation. Sciama [310] (1991) says that: "One sees occasional claims that some varieties of superstring theory lead naturally to the vanishing of the cosmological constant, but no agreement has yet been reached on this point. My view is in §2.6. Some recent work includes the papers mentioned below.

Baum [30] (1984) imposes a minimum action on a scalar field coupled to classical gravity and find zero effective cosmological constant without fine tuning, as well as a new mechanism for symmetry breaking, see §2.10.

Carvahlo *et al* [376] (1992) argue for the first time that the dimensional argument of Chen and Wu [75] is not enough to fix the time dependence of the Cosmological Constant. They claim to show that a similar argument leads to Λ proportional to H^2. The cosmological consequences of this new possibility are discussed in detail. In particular, they claim to show that as opposed to the Chen and Wu model, this new dependence solve the age problem.

Moffat [249] (1994) presents a dynamical model of the decaying vacuum energy, which is based on Jordan-Brans-Dicke theory with a scalar field ϕ. The solution of an evolutionary differential equation for the scalar field ϕ drives the vacuum energy towards a cosmological constant at the present epoch that can give for the age of the universe, $t_0 \sim 13.5$ Gyr for $\Omega_0 = 1$, which is consistent with the age of globular clusters.

Barrow and Dąbrowski [26] (1995) note that if a positive cosmological constant exists then these oscillations will eventually cease and be replaced by an era of expansion which will continue unless the cosmological constant is associated with a form of vacuum energy that ultimately decays away.

Krauss and Turner [206] (1995) claim that a diverse set of observations now compellingly suggest that Universe possesses a nonzero cosmological constant. They say that in the context of quantum-field theory a cosmological constant corresponds to the energy density of the vacuum, and the wanted value for the cosmological constant corresponds to a very tiny vacuum energy density. They discuss future observational tests for a cosmological constant as well as the fundamental theoretical challenges—and opportunities—that this poses for particle physics and for extending our understanding of the evolution of the Universe back to the earliest moments.

Singh [318] (1995) notes that increasing improvements in the independent determinations of the Hubble constant and the age of the universe now seem to indicate that a small non-vanishing cosmological constant is needed to make the two independent observations consistent with each other. The cosmological constant can be physically interpreted as due to the vacuum energy of quantized fields. To make the cosmological observations consistent with each other he suggests that a vacuum energy density, $\rho_v \sim (10^{-3} eV)^4$ is needed today (in the units $\hbar = c = k = 1$ sometimes called cosmological units). It is argued in his article that such a vacuum energy density is natural in the context of phase transitions linked to massive neutrinos. In fact, the neutrino masses required to provide the right vacuum energy scale to remove the age verses Hubble constant discrepancy are consistent with those required to solve the solar neutrino problem by the MSW mechanism.

Lima [390] (1996) studies the thermodynamic behaviour of a decaying $\Lambda - term$ coupled to a relativistic simple fluid. Using the covariant approach for nonequilibrium thermodynamics, he shows that if the specific entropy per particle is conserved (adiabatic decay), some equilibrium relations are preserved. In particular, if the vacuum decay adiabatically in photons, the Planckian form of the **CBR** spectrum is also preserved in the course of the evolution. The photon spectrum is deduced in the Appendix.

Adler *et al* [7] (1997) notes that recent work has shown that complex quantum field theory emerges as a statistical mechanical approximation to an underlying noncommutative operator dynamics based on a total trace action. In this dynamics, scale invariance of the trace action becomes the statement $0 = \Re Tr T_\mu^\mu$, with $T_{\mu\nu}$ the operator stress energy tensor, and with Tr the trace over the underlying Hilbert space. They show that this condition implies the vanishing of the cosmological constant and vacuum energy in the emergent quantum field theory. However, they say that since the scale invariance condition does not require the operator T_μ^μ to vanish, the spontaneous breakdown of scale invariance is still permitted.

Coble *et al* [82] (1997) note that models of structure formation with a cosmological constant Λ provide a good fit to the observed power spectrum of galaxy clustering. However, these models suffer from several problems. Theoretically, it is difficult to understand why the cosmological constant is so small in Planck units. Observationally, while the power spectra of cold dark matter plus Λ models have approximately the right shape, the COBE-normalized amplitude for a scale invariant spectrum is too high, requiring galaxies to be anti-biased relative to the mass distribution. Attempts to address the first problem have led to models in which a dynamical field supplies the vacuum energy, which is thereby determined by fundamental physics scales. They explore the implications of such dynamical Λ models for the formation of large-scale structure. They find that there are dynamical models for which the amplitude of the COBE-normalized spectrum matches the observations. They also calculate the cosmic microwave background anisotropies in these models and show that the angular power spectra are distinguishable from those of standard cosmological constant models.

Dolgov [103] (1997) analyses the cosmological evolution of free massless vector or tensor (but not gauge) fields minimally coupled to gravity. He shows that there are some unstable solutions for these fields in De Sitter background. The back reaction of the energy-momentum tensor of such solutions to the original cosmological constant exactly cancels the latter and the expansion regime changes from the exponential to the power law one. In contrast to the adjustment mechanism realized by a scalar field the gravitational coupling constant in this model is time-independent and the resulting cosmology may resemble the realistic one.

Guendelman and Kaganovich [157] (1997) claim to have shown that the principle of nongravitating vacuum energy, when formulated in the first order formalism, solves the cosmological constant problem. The most appealing formulation of the theory displays a local symmetry associated with the arbitrariness of the measure of integration. This can be motivated by thinking of this theory as a direct coupling of physical degrees of freedom with a "space - filling brane" and in this case such local symmetry is related to space-filling brane gauge invariance. The model is formulated in the first order formalism using the metric and the connection as independent dynamical variables. An additional symmetry (Einstein - Kaufman symmetry) allows to elimination of the torsion which appears due to the introduction of the new measure of integration. The most successful model that implements these ideas is realized in a six or higher dimensional spacetime. The compactification of extra dimensions into a sphere gives the possibility of generating scalar masses and potentials, gauge fields and fermionic masses. It turns out that remaining four dimensional spacetime must have effective zero cosmological constant.

Alvarenga and Lemos [12] (1998) regard the vacuum as a perfect fluid with equation of state $p = -\rho$, de Sitter's cosmological model is quantized. Their treatment differs from previous ones in that it endows the vacuum with dynamical degrees of freedom. Instead of being postulated from the start, the cosmological constant arises from the degrees of freedom of the vacuum regarded as a dynamical entity, and a time variable can be naturally introduced. Taking the scale factor as the sole degree of freedom of the gravitational field, stationary and wave packet solutions to the Wheeler-DeWitt equation are found. It turns out that states of the Universe with a definite value of the cosmological constant do not exist. For the wave packets investigated, quantum effects are noticeable only for small values of the scale factor, a classical regime being attained at asymptotically large times.

Ries *et al* [281] (1998) present observations of 10 type Ia supernovae (SNe Ia) between $0.16 < z < 0.62$. With previous data from their high-Z supernova search team, this expanded set of 16 high-redshift supernovae and 34 nearby supernovae are used to place constraints on the Hubble constant (H_0), the mass density (Ω_M), the cosmological constant (Ω_Λ), the deceleration parameter (q_0), and the dynamical age of the Universe (t_0). The distances of the high-redshift SNe Ia are, on average, 10% to 15% farther than expected in a low mass density ($\Omega_M = 0.2$) Universe without a cosmological constant. Different light curve fitting methods, SN Ia subsamples, and prior constraints unanimously favor eternally expanding models with positive cosmological constant (i.e., $\Omega_\Lambda > 0$) and a current acceleration of the expansion (i.e., $q_0 < 0$). With no prior constraint on mass density other than $\Omega_M > 0$, the spectroscopically confirmed SNe Ia are consistent with $q_0 < 0$ at the 2.8σ and 3.9σ confidence levels, and with $\Omega_\Lambda > 0$ at the 3.0σ and 4.0σ confidence levels, for two fitting methods respectively. Fixing a "minimal" mass density, $\Omega_M = 0.2$, results in the weakest detection, $\Omega_\Lambda > 0$ at the 3.0σ confidence level. For a flat-Universe prior ($\Omega_M + \Omega_\Lambda = 1$), the spectroscopically confirmed SNe Ia require $\Omega_\Lambda > 0$ at 7σ and 9σ level for the two fitting methods. A Universe closed by ordinary matter (i.e., $\Omega_M = 1$) is ruled out at the 7σ to 8σ level. They estimate the size of systematic errors, including evolution, extinction, sample selection bias, local flows, gravitational lensing, and sample contamination. Presently, none of these effects reconciles the data with $\Omega_\Lambda = 0$ and $q_0 > 0$.

Tegmark [335] (1998) describes constraints on a "standard" 8 parameter open cold dark matter (CDM) model from the most recent CMB and SN1a data. His parameters are the densities of CDM, baryons, vacuum energy and curvature, the reionization optical depth, and the normalization and tilt for both scalar and tensor fluctuations. He finds that although the possibility of reionization and gravity waves substantially weakens the constraints on CDM and baryon density, tilt, Hubble constant and curvature, allowing e.g. a closed Universe, open models with vanishing cosmological constant are still strongly disfavored.

Adams *et al* [4] (1999) explore possible effects of vacuum energy on the evolution of black holes. If the universe contains a cosmological constant, and if black holes can absorb energy from the vacuum, then black hole evaporation could be greatly suppressed. For the magnitude of the cosmological constant suggested by current observations, black holes larger than $\sim 4 \times 10^{24}$ g would accrete energy rather than evaporate. In this scenario, all stellar and supermassive black holes would grow with time until they reach a maximum mass scale of $\sim 6 \times 10^{55}$ g, comparable to the mass contained within the present day cosmological horizon.

Alcaniz and Lima [9] (1999) notes that the ages of two old galaxies (53W091, 53W069) at high redshifts are used to constrain the value of the cosmological constant in a flat universe (ΛCDM) and the density parameter Ω_M in Friedmann-Robertson-Walker (FRW) models with no Λ-term. In the case of ΛCDM models, the quoted galaxies yield two lower limits for the vacuum energy density parameter, $\Omega_\Lambda \geq 0.42$ and $\Omega_\Lambda \geq 0.5$, respectively. Although compatible with the limits from statistics of gravitational lensing (SGL) and cosmic microwave background (CMB), these lower bounds are more stringent than the ones recently determined using SNe Ia as standard candles. For matter dominated universes ($\Omega_\Lambda = 0$), the existence of these galaxies imply that the universe is open with the matter density parameter constrained by $\Omega_M \leq 0.45$ and $\Omega_M \leq 0.37$, respectively. In particular, these results disagree completely with the analysis of field galaxies which gives a lower limit $\Omega_M \geq 0.40$.

Bond and Jaffe [46] (1999) use Bayesian analysis methods to determine what current CMB and $CMB+LSS$ data imply for inflation-based Gaussian fluctuations in tilted ΛCDM, $\Lambda hCDM$ and $oCDM$ model sequences with cosmological age 11-15 Gyears, consisting of mixtures of bayrons, cold 'c' (and possibly hot 'h') dark matter, vacuum energy 'Λ', and curvature energy 'o' in open cosmologies.

Burdyuzha *et al* [61] (1999) discuss the problem of the physical nature of the cosmological constant genesis. They say that this problem can't be solved in terms of current quantum field theory which operates with Higgs and non-perturbative vacuum condensates and takes into account the changes of these condensates during relativistic phase transitions. They also say that the problem can't be completely solved also in terms of the conventional global quantum theory: Wheeler-DeWitt quantum geometrodynamics does not describe the evolution of the Universe in time (RPT in particular). They have investigated this problem in the context of energies density of different vacuum subsystems characteristic scales of which pervade all energetic scale of the Universe. At first the phemenological solution of the cosmological constant problem and then the hypothesis about the possible structure of a new global quantum theory are proposed. The main feature of this theory is the irreversible evolution of geometry and vacuum condensates in time in the regime of their self organization. The transformation of the cosmological constant in dynamical variable is inevitably.

Jackson [182] (1999) notes that recent observations suggest that Hubble's constant is large, to the extent that the oldest stars appear to have ages which are greater than the Hubble time, and that the Hubble expansion is slowing down, so that according to conventional cosmology the age of the Universe is less than the Hubble time. He introduces the concepts of weak and strong age crises (respectively $t_0 < 1/H_0$ but longer than the age inferred from some lower limit on q_0, and $t_0 > 1/H_0$ and $q_0 > 0$) are introduced. These observations are reconciled in models which are dynamically dominated by a homogeneous scalar field, corresponding to an ultra-light boson whose Compton wavelength is of the same order as the Hubble radius. Two such models are considered, an open one with vacuum energy comprising a conventional cosmological term and a scalar field component, and a flat one with a scalar component only, aimed respectively at weak and strong age crises. Both models suggest that anti-gravity plays a significant role in the evolution of the Universe.

Krauss [205] (1999) says that there are two contenders to explain changes in the expansion rate of the Universe, one is the cosmological constant; he seems reluctant to say what the other is, but it appears to be some form of dark matter.

Pavsic [270] (1999) studies the harmonic oscillator in pseudo euclidean space. A straightforward procedure reveals that although such a system may have negative energy, it is stable. In the quantized theory the vacuum state has to be suitably defined and then the zero-point energy corresponding to a positive-signature component is canceled by the one corresponding to a negative-signature component. This principle is then applied to a system of scalar fields. The metric in the space of fields is assumed to have signature (+ + + ... - - -) and it is shown that the vacuum energy, and consequently the cosmological constant, are then exactly zero. The theory also predicts the existence of stable, negative energy field excitations (the so called "exotic matter") which are sources of repulsive gravitational fields, necessary for construction of the time machines and Alcubierre's hyperfast warp drive.

Sahni [302] (1999) notes that the close relationship between the cosmological constant and the vacuum has been emphasized in the past by Zeldovich amongst others. Sahni briefly discusses different approaches to the cosmological constant issue including the possibility that it could be generated by vacuum polarization in a static universe. Fresh possibilities occur in an expanding universe. An inflationary universe generically leads to particle creation from the vacuum, the nature and extent of particle production depending upon the mass of the field and its coupling to gravity. For ultra-light, non-minimally coupled scalar fields, particle production can be large and the resulting vacuum energy-momentum tensor will have the form of a cosmological constant. The inflationary scenario therefore, could give rise to a universe that is both flat and Λ-dominated, in agreement with observation.

Sivaram [321] (1999) notes that recent attempts have been made to link vacuum zero-point fields (ZPF) with a non-zero cosmological constant (Λ), which is now treated as a cosmological free variable to be determined by observation. he claims that in another recent paper, Λ is related to a graviton mass. He says that his is shown to be incorrect. Flat space propagators for both massless and massive spin-2 particles can be written (independently of gravity) in the context of flat space wave equations; however, they do not correspond to full general relativity with a graviton mass.

Tkach *et al* [338] (1999) present a new hidden symmetry in gravity for the scale factor in the FRW model, for $k = 0$. This exact symmetry vanishes the cosmological constant. They interpret this hidden symmetry as a dual symmetry in the sense that appears in the string theory.

Vishwakarma [348] (1999) investigate some Friedmann models in which Λ varies as ρ, by modifying the Chen and Wu ansatz. In order to test the consistency of the models with observations, he studies the angular size - redshift relation for 256 ultracompact radio sources selected by Jackson and Dodgson. The angular sizes of these sources were determined by using very long-baseline interferometry in order to avoid any evolutionary effects. The models fit to the data very well and demand an accelerating universe with a positive cosmological constant. Open, flat as well as closed models are almost equally probable, though the open model provides comparatively a better fit to the data. The models are found to have intermediate density and postulate the existence of dark matter, though not as high as in the canonical Einstein-deSitter model.

Axenides *et al* [21] (2000) note that Newton's law gets modified in the presence of a cosmological constant by a small repulsive term (antigravity) that is proportional to the distance. Assuming a value of the cosmological constant consistent with the recent SnIa data ($\Lambda \sim eq 10^{-52} m^{-2}$) they investigate the significance of this term on various astrophysical scales. They find that on galactic scales or smaller (less than a few tens of kpc) the dynamical effects of the vacuum energy are negligible by several orders of magnitude. On scales of 1Mpc or larger however they find that vacuum energy can significantly affect the dynamics. For example they shows that the velocity data in the Local Group of galaxies correspond to galactic masses increased by 35% in the presence of vacuum energy. The effect is even more important on larger low density systems like clusters of galaxies or superclusters.

Buosso and Polchinski [50] (2000) note that a four-form gauge flux makes a variable contribution to the cosmological constant. This has often been assumed to take continuous values, but they argue that it has a generalized Dirac quantization condition. For a single flux the steps are much larger than the observational limit, but they show that with multiple fluxes the allowed values can form a sufficiently dense 'discretuum'. Multiple fluxes generally arise in M theory compactifications on manifolds with non-trivial three-cycles. In theories with large extra dimensions a few four-forms suffice; otherwise of order 100 are needed. Starting from generic initial conditions, the repeated nucleation of membranes dynamically generates regions with a cosmological constant in the observational range. Entropy and density perturbations can be produced.

Elizalde [112] (2000) introduce a simple model in which the cosmological constant is interpreted as a true Casimir effect on a scalar field filling the universe (e.g. $\mathbf{R} \times \mathbf{T}^p \times \mathbf{T}^q$, $\mathbf{R} \times \mathbf{T}^p \times \mathbf{S}^q, ...$). The effect is driven by compactifying boundary conditions imposed on some of the coordinates, associated both with large and small scales. The very small, but non zero-value of the cosmological constant obtained from recent astrophysical observations can be perfectly matched with the results coming from the model, by playing just with the numbers of actually compactified ordinary and tiny dimensions, and being the compactification radius (for the last) in the range $(1 - 10^3) l_{Pl}$, where l_{Pl} is the Planck length. This corresponds to solving, in a way, what has been termed by Weinberg the *new* cosmological constant problem. Moreover, a marginally closed universe is favored by the model, again in coincidence with independent analysis of the observational results.

Fiziev [127] (2000) notes that in the framework of a model of minimal of dilatonic gravity (MDG) with cosmological potential that he considers: the relations of MDG with nonlinear gravity and string theory; natural cosmological units, defined by cosmological constant; the properties of cosmological factor, derived from solar system and Earth-surface gravitational experiments; universal anti-gravitational interactions, induced by positive cosmological constant and by Nordtveld effect; a new formulation of cosmological constant problem using the ratio of introduced cosmological action and Planck constant $\sim 10^{122}$;

inverse cosmological problem: to find cosmological potential which yields given evolution of the RW Universe; and comment other general properties of MDG.

Freedman [131] (2000) notes that rapid progress has been made recently toward the measurement of cosmological parameters. Still, there are areas remaining where future progress will be relatively slow and difficult, and where further attention is needed. In this review, the status of measurements of the matter density, the vacuum energy density or cosmological constant, the Hubble constant, and ages of the oldest measured objects are summarized. Many recent, independent dynamical measurements are yielding a low value for the matter density of about 1/3 the critical density. New evidence from type Ia supernovae suggests that the vacuum energy density may be non-zero. Many recent Hubble constant measurements appear to be converging in the range of 65-75 km/sec/Mpc. Eliminating systematic errors lies at the heart of accurate measurements for all of these parameters; as a result, a wide range of cosmological parameter space is currently still open. Fortunately, the prospects for accurately measuring cosmological parameters continue to increase.

Futamase and Yoshida [137] (2000) propose a possible measurement of the variability of the vacuum energy, perhaps here meaning some time dependent cosmological constant, using strong gravitational lensing. As an example they take an Einstein cross lens HST 14176+5226 and show that the measurement of the velocity dispersion with the accuracy of $\pm$ 5km/sec determines the density parameter with the accuracy of order 0.1, and it clarifies the existence of the vacuum energy as well as its variability with redshift.

Guilini and Straumann [147] (2000) note that the principles of general relativity allow for a non-vanishing cosmological constant, which can possibly be interpreted at least partially in terms of quantum-fluctuations of matter fields. Depending on sign and magnitude it can cause accelerated or decelerated expansion at certain stages of cosmic evolution. Recent observations in cosmology seem to indicate that we presently live in an accelerated phase. They recall the history and fundamental issues connected with the cosmological constant and then discuss present evidences for a positive value, which causes the accelerated expansion.

Kakushadze [190] (2000) considers a recent proposal to solve the cosmological constant problem within the context of brane world scenarios with infinite volume extra dimensions. In such theories bulk can be supersymmetric even if brane supersymmetry is completely broken. The bulk cosmological constant can therefore naturally be zero. Since the volume of the extra dimensions is infinite, it might appear that at large distances one would measure the bulk cosmological constant which vanishes. He points out a caveat in this argument. In particular, he uses a concrete model, which is a generalization of the Dvali-Gabadadze-Porrati model, to argue that in the presence of non-zero brane cosmological constant at large distances such a theory might become effectively four dimensional. This is due to a mass gap in the spectrum of bulk graviton modes. In fact, the corresponding distance scale is set precisely by the brane cosmological constant. This phenomenon appears to be responsible for the fact that bulk supersymmetry does not actually protect the brane cosmological constant.

Kehagias and Tamvakis [195] (2000) discuss the four-dimensional cosmological constant problem in a five-dimensional setting. A scalar field coupled to the SM forms dynamically a smooth brane with four-dimensional Poincare invariance, independently of SM physics. In this respect, their solution might be regarded as a self-tuning solution, free of any singularities and fine-tuning problems.

Parker and Raval [268] (2000) propose a new model where nonperturbative vacuum contributions to the effective action of a free quantized massive scalar field lead to a cosmological solution in which the scalar curvature becomes constant after a time t_j (when the redshift $z \sim 1$) that depends on the mass of the scalar field and its curvature coupling. This spatially-flat solution implies an accelerating universe at the present time and gives a good one-parameter fit to high-redshift Type Ia supernovae (SNe-Ia) data, and the present age and energy density of the universe. Here they show that the imaginary part of the nonperturbative curvature term that causes the cosmological acceleration, implies a particle production rate that agrees with predictions of other methods and extends them to non-zero mass fields. The particle production rate is very small after the transition and is not expected to alter the nature of the cosmological solution. They also show that the equation of state of our model undergoes a transition at t_j from an equation of state dominated by non-relativistic pressureless matter (without a cosmological constant) to an effective equation of state of mixed radiation and cosmological constant, and they derive the equation of state of the vacuum. Finally, they explain why nonperturbative vacuum effects of this ultra low mass particle do not significantly change standard early universe cosmology.

Straumann [330] (2000) explains why the (effective) cosmological constant is expected to obtain contributions from short distance physics, corresponding to an energy at least as large as the Fermi scale, after a short history of the Λ-term. The actual tiny value of the cosmological constant by particle physics standards represents, therefore, one of the deepest mysteries of present-day fundamental physics. Recent proposals of an approach to the cosmological constant problem which make use of (large) extra dimensions are briefly discussed. Cosmological models with a dynamical Λ, which attempt to avoid the disturbing cosmic coincidence problem, are also reviewed.

The *principle of holography* is that only the boundary of a system needs to be considered, because the surface determines the dynamics of the system; effectively this means that it is only necessary to work in one less dimension. Thomas [336] (2000) argues that gravitational holography renders the cosmological constant stable against divergent quantum corrections. This provides a technically natural solution to the cosmological constant problem. A natural solution can follow from a symmetry of the action. Evidence for quantum stability of the cosmological constant is illustrated in a number of examples including, bulk descriptions in terms of delocalized degrees of freedom, boundary screen descriptions on stretched horizons, and non-supersymmetric conformal field theories as dual descriptions of anti-de Sitter space. In an expanding universe, holographic quantum contributions to the stress-energy tensor are argued to be at most of order the energy density of the dominant matter component.

Tye and Wasserman [342] (2000) consider a model with two parallel (positive tension) 3-branes separated by a distance L in 5-dimensional spacetime. If the interbrane space is anti-deSitter, or is not precisely anti-deSitter but contains no event horizons, the effective 4-dimensional cosmological constant seen by observers on one of the branes (chosen to be the visible brane) becomes exponentially small as L grows large.

Chen [400] (2001) provides a new way of looking at the vacuum which has implications for the cosmological constant, see §2.1 above.

Volovik [412] (2001) discusses condensed matter examples, in which the effective gravity appears in the low-energy corner as one of the collective modes of quantum vacuum, provide a possible answer to the question, why the vacuum energy is so small. He says that this answer comes from the fundamental "trans-Planckian" physics of quantum liquids. In the effective theory of the low energy degrees of freedom the vacuum energy density is proportional to the fourth power of the corresponding "Planck" energy appropriate for this effective theory. However, from the exact "Theory of Everything" of the quantum liquid it follows that its vacuum energy density is exactly zero without fine tuning, if: there are no external forces acting on the liquid; there are no quasiparticles which serve as matter; no spacetime curvature; and no boundaries which give rise to the Casimir effect. Each of these four factors perturbs the vacuum state and induces the nonzero value of the vacuum energy density of order of energy density of the perturbation. This is the reason, why one must expect that in each epoch the vacuum energy density is of order of matter density of the Universe, or/and of its curvature, or/and of the energy density of smooth component – the quintessence.

Gurzadyan and Xue [405] (2001) present their view on the problem of the cosmological constant and vacuum energy. They point out that only relevant modes of the vacuum fluctuation, whose wavelengths are conditioned by the size, homogeneity, geometry and topology of the present Universe, contribute to the cosmological constant. As a result, the cosmological constant is expressed in terms of the size of the Universe and the three fundamental constants: the velocity of light, Planck and Newton gravitational constants. They say that its present value remarkably agrees with the recent observations and its dependence on the size of the Universe confronts with observations.

4.2 The Anthropic Principle.

Weinberg [353] considers the *anthropic principle*. He says that in several cosmological theories the observed big bang is just one member of an ensemble. The ensemble might consist of different expanding regions at different times and locations in the same spacetime, see also Vilenkin [344] (1983) and Linde [228] (1986), or in different terms in the "wave function of the Universe", see Baum [30] (1984). If the vacuum energy density ρ_Λ varies among the different members of this ensemble, then the value observed by any species of astronomers will be conditioned by the necessity that this value of ρ_Λ should be suitable for the evolution of intelligent life. Perhaps one can think of the anthropic principle as only allowing certain sorts of vacuum energy. The anthropic principle has also been reviewed by Gale [142] (1981). Garriga and Vilenkin [144] (2000) argue that the anthropic principle is necessary to explain the time coincidence of a supposed epoch of galaxy formation and an epoch dominated by the cosmological constant. One can also think of the anthropic principle in terms of a given set of conditions or boundaries. Only universes (more accurately parts of the Universe) which create conditions for observers to occur can be measured. One can ask when else one could apply a set of conditions. *Two* places occur. *One* at or near the beginning of the universe, where a set of initial conditions could be choosen. The *other* at a late time, where it is not immediately clear what sort of conditions one would want to apply. Most cosmological models are either too hot or cold at late times and one might invoke a "Happy Ending Principle" that the temperature is in the region for life to continue to exist. Some recent work on the anthropic principle includes the papers mentioned below.

Sivaram [320] (1999) says that an impressive variety of recent observations which include luminosity evolutions of high redshift supernovae strongly suggest that the cosmological constant Λ is not zero. Even though the Λ-term might dominate cosmic dynamics at the present epoch, such a value for the vacuum energy is actually unnaturally small. The difficulty finding a suitable explanation (based upon fundamental physics) for such a small residue value for the cosmological term has led several authors to resort to an anthropic explanation of its existence. Sivaram presents a few examples which invoke phase transitions in the early universe involving strong or electroweak interactions to show how the cosmic term of the correct observed magnitude can arise from fundamental physics involving gravity.

Banks *et al* (2000) are motivated by recent work of Bousso and Polchinski [25] (2000), they study theories which explain the small value of the cosmological constant using the anthropic principle. They argue that simultaneous solution of the gauge hierarchy problem is a strong constraint on any such theory. They exhibit *three* classes of models which satisfy these constraints. The *first* is a version of the BP model with precisely two large dimensions. The *second* involves 6-branes and antibranes wrapped on supersymmetric 3-cycles of Calabi-Yau manifolds, and the *third* is a version of the irrational axion model. All of them have possible problems in explaining the size of microwave background fluctuations. They also find that most models of this type predict that all constants in the low energy Lagrangian, as well as the gauge groups and representation content, are chosen from an ensemble and cannot be uniquely determined from the fundamental theory. In their opinion, this significantly reduces the appeal of this kind of solution of the cosmological constant problem. On the other hand, they argue that the vacuum selection problem of string theory might plausibly have an anthropic, cosmological solution.

Donoghue [104] (2000) notes that one way that an anthropic selection mechanism might be manifest in a physical theory involves multiple domains in the universe with different values of the physical parameters. If this mechanism is to be relevant for understanding the small observed value of the cosmological constant, it might involve a mechanism by which some contributions to the cosmological constant can be fixed at a continuous range of values in the different domains. He studies the properties of four possible mechanisms, including the possibility of the Hubble damping of a scalar field with an extremely flat potential. Another interesting possibility involves fixed random values of non-dynamical form fields, and a cosmological mechanism is suggested. This case in addition raises the possibility of anthropic selection of other parameters. He discusses further requirements needed for a consistent cosmology.

Melchiorri and Griffiths [393] (2000) analyse boomarang data and find the Universe is very close to flat.

4.3 Quintessence.

Quintessence was introduced by Peebles and Ratra [271] (1988), see also Weinberg [353] §2 (2000). The idea is that the cosmological constant is small because the Universe is old. How instead of a cosmological constant vacuum energy might be manifest as a scalar fields is described in §2.1¶3. This scalar field perhaps can be thought of as a time dependent cosmological constant. Some recent work on quintessence includes the papers below.

Zlatev *et al* [368] (1998) notes that recent observations suggest that a large fraction of the energy density of the universe has negative pressure. One explanation is vacuum energy density; another is quintessence in the form of a scalar field slowly evolving down a potential. In either case, a key problem is to explain why the energy density nearly coincides with the matter density today. The densities decrease at different rates as the universe expands, so coincidence today appears to require that their ratio be set to a specific, infinitesimal value in the early universe. In this paper, they introduce the notion of a "tracker field", a form of quintessence, and show how it may explain the coincidence, adding new motivation for the quintessence scenario.

Bento and Bertolami [38] (1999) study the possibility that the vacuum energy density of scalar and internal-space gauge fields arising from the process of dimensional reduction of higher dimensional gravity theories plays the role of quintessence. They show that, for the multidimensional Einstein-Yang-Mills system compactified on a $R \times S^3 \times S^d$ topology, there are classically stable solutions such that the observed accelerated expansion of the Universe at present can be accounted for without upsetting structure formation scenarios or violating observational bounds on the vacuum energy density.

Chiba [77] (1999) notes that dynamical vacuum energy or quintessence, a slowly varying and spatially inhomogeneous component of the energy density with negative pressure, is currently consistent with the observational data. One potential difficulty with the idea of quintessence is that couplings to ordinary matter should be strongly suppressed so as not to lead to observable time variations of the constants of nature. He further explores the possibility of an explicit coupling between the quintessence field and the curvature. Since such a scalar field gives rise to another gravity force of long range ($\sim H_0^{-1}$), the solar system experiments put a constraint on the non-minimal coupling: $|\xi| \sim 10^{-2}$.

Chimento *et al* [78] (2000) note that the combined effect of a dissipative fluid and quintessence energy can simultaneously drive an accelerated expansion phase at the present time and solve the coincidence problem of our current Universe. A solution compatible with the observed cosmic acceleration is succinctly presented. In Chimento *et al* [386] (2000) they show that the combination of a fluid with a bulk dissipative pressure and quintessence matter can simultaneously drive an accelerated expansion phase and solve the coincidence problem of our current Universe. They then study some scenarios compatible with the observed cosmic acceleration.

Hebecker and Wetterich [169] (2000) formulate conditions for the naturalness of cosmological quintessence scenarios. They take the quintessence lagrangian is taken to be the sum of a simple exponential potential and a non-canonical kinetic term. This parameterization covers most variants of quintessence and makes the naturalness conditions particularly transparent. Several "natural" scalar models lead, for the present cosmological era, to a large fraction of homogeneous dark energy density and an acceleration of the scale factor as suggested by observation.

Amendola [14] (1999) notes that a new component of the cosmic medium, a light scalar field or "quintessence ", has been proposed recently to explain cosmic acceleration with a dynamical cosmological constant. Such a field is expected to be coupled explicitly to ordinary matter, unless some unknown symmetry prevents it. He investigates the cosmological consequences of such a coupled quintessence (CQ) model, assuming an exponential potential and a linear coupling. This model is conformally equivalent to Brans-Dicke Lagrangians with power-law potential. He evaluates the density perturbations on the cosmic microwave background and on the galaxy distribution at the present and derive bounds on the coupling constant from the comparison with observational data. A novel feature of CQ is that during the matter dominated era the scalar field has a finite and almost constant energy density. This epoch, denoted as ϕMDE, is responsible of several differences with respect to uncoupled quintessence: the multipole spectrum of the microwave background is tilted at large angles, the acoustic peaks are shifted, their amplitude is changed, and the present $8\mathrm{Mpc}/h$ density variance is diminished. The present data constrain the dimensionless coupling constant to $|\beta| \leq 0.1$.

Wiltshire [358] (2000) notes that quintessence models with a dark energy generated by pseudo Nambu-Goldstone bosons provide a natural framework in which to test the possibility that type Ia supernovae luminosity distance measurements are at least partially due to an evolution of the sources, since these models can have parameter values for which the expansion of the Universe is decelerating as well as values for which it is accelerating, while being spatially flat in all cases and allowing for a low density of clumped matter. The results of a recent investigation by Wiltshire of current observational bounds which allow for SNe Ia source evolution are discussed. He finds that models with source evolution still favour cosmologies with an appreciable amount of acceleration in the recent past, but that the region of parameter space which is most favoured shifts significantly.

4.4 Inflation.

The idea here is that the scale factor R of Robertson-Walker spacetime takes an exponential form $R = \exp(at)$ which is supposed to correspond to the size of the Universe increasing exponentially. deSitter spacetime can be caste in $R = \exp(at)$ form, and deSitter spacetime is static; however in the inflation picture the Universe is expanding exponentially, the resolution of this is apparently in Schrödinger [308] (1956). Pollock and Dahdev [276] (1989), discuss how induced gravity, see §2.15 fits in with inflation. There is a lot written on inflation, here about 15 recent papers are mentioned.

Knox and Turner [200] (1993) present a simple model for slow-rollover inflation where the vacuum energy that drives inflation is of the order of G_F^{-2}; unlike most models, the conversion of vacuum energy to radiation ("reheating") is moderately efficient. The scalar field responsible for inflation is a standard-model singlet, develops a vacuum expectation value of the order of $4 \times 10^6 GeV$, has a mass of order $1GeV$, and can play a role in electroweak phenomena.

Spokoiny [325] (1993) shows that it is possible to realize an inflationary scenario even without conversion of the false vacuum energy to radiation. Such cosmological models have a deflationary stage in which Ha^2 is decreasing and radiation produced by particle creation in an expanding Universe becomes dominant. The preceding inflationary stage ends since the inflaton potential becomes steep. False vacuum energy is finally (partly) converted to the inflaton kinetic energy, the potential energy rapidly decreases and the Universe comes to the deflationary stage with a scale factor $a(t) \propto t^{1/3}$. Basic features and observational consequences of this scenario are indicated.

Copeland *et al* [87] (1994) investigate chaotic inflation models with two scalar fields, such that one field (the inflaton) rolls while the other is trapped in a false vacuum state. The false vacuum becomes unstable when the inflaton field falls below some critical value, and a first or second order transition to the true vacuum ensues. Particular attention is paid to Linde's second-order 'Hybrid Inflation'; with the false vacuum dominating, inflation differs from the usual true vacuum case both in its cosmology and in its relation to particle physics. The spectral index of the adiabatic density perturbation can be very close to 1, or it can be around ten percent higher. The energy scale at the end of inflation can be anywhere between 10^{16} GeV and 10^{11} GeV, though reheating is prompt so the reheat temperature can't be far below 10^{11} GeV. Topological defects are almost inevitably produced at the end of inflation, and if the inflationary energy scale is near its upper limit they can have significant effects. Because false vacuum inflation occurs with the inflaton field far below the Planck scale, it is easier to implement in the context of supergravity than standard chaotic inflation. That the inflaton mass is small compared with the inflationary Hubble parameter is still a problem for generic supergravity theories, but remarkably this can be avoided in a natural way for a class of supergravity models which follow from orbifold compactification of superstrings. This opens up the prospect of a truly realistic, superstring.

Gaillard *et al* [139] (1995) note that supersymmetry is generally broken by the non-vanishing vacuum energy density present during inflation. In supergravity models, such a source of supersymmetry breaking typically makes a contribution to scalar masses of order $\tilde{m}^2 \sim H^2$, where $H^2 \sim V/M_P^2$ is the Hubble parameter during inflation. They show that in supergravity models which possess a Heisenberg symmetry, supersymmetry breaking makes no contribution to scalar masses, leaving supersymmetric flat directions flat at tree-level. One-loop corrections in general lift the flat directions, but naturally give small negative squared masses $\sim -g^2 H^2/(4\pi)^2$ for all flat directions that do not involve the stop. No-scale supergravity of the SU(N,1) type and the untwisted sectors from orbifold compactifications are special cases of this general set of models. They point out the importance of the preservation of flat directions for baryogenesis.

Gilbert [146] (1995) examines inflationary potentials which produce power-law density perturbations. The models derived are dominated by a false vacuum energy at late times and inflate indefinitely. This paper also examines the effects on the fluctuation spectrum of reducing the potential to end inflation. Small reductions of the potential result in little change to the perturbation spectrum. The effect of a large reduction, however, is to change the sign of the slope of the fluctuation spectrum.

Berera [39] (1996) notes that in the standard picture, the inflationary universe is in a supercooled state which ends with a short time, large scale reheating period, after which the universe goes into a radiation dominated stage. Here he proposes an alternative in which the radiation energy density smoothly decreases during an inflation-like stage and with no discontinuity enters the subsequent radiation dominated stage. The scale factor is calculated from standard Friedmann cosmology in the presence of both radiation and vacuum energy density. A large class of solutions confirm the above identified regime of non-reheating inflation-like behavior for observationally consistent expansion factors and not too large a drop in the radiation energy density. One dynamical realization of such inflation without reheating is from warm inflation type scenarios. However the solutions found here are properties of the Einstein equations with generality beyond slow-roll inflation scenarios. The solutions also can be continuously interpolated from the non-reheating type behavior to the standard supercooled limit of exponential expansion, thus giving all intermediate inflation-like behavior between these two extremes. The temperature of the universe and the expansion factor are calculated for various cases. Implications for baryongenesis are discussed. This non-reheating, inflation-like regime also appears to have some natural features for a universe that is between nearly flat and open.

Boyananovsky *et al* (1996) [51] analyzes the phenomenon of preheating, i.e. explosive particle production due to parametric amplification of quantum fluctuations in the unbroken case, or spinodal instabilities in the broken phase, using the Minkowski space $O(N)$ vector model in the large N limit to study the non-perturbative issues involved. They give analytic results for weak couplings and times short compared to the time at which the fluctuations become of the same order as the tree level, as well as numerical results including the full backreaction. In the case where the symmetry is unbroken, the analytic results agree spectacularly well with the numerical ones in their common domain of validity. In the broken symmetry case, slow roll initial conditions from the unstable minimum at the origin, give rise to a new and unexpected phenomenon: the dynamical relaxation of the vacuum energy. That is, particles are abundantly produced at the expense of the quantum vacuum energy while the zero mode comes back to almost its initial value. In both cases we obtain analytically and numerically the equation of state which turns to be written in terms of an effective polytropic index that interpolates between vacuum and radiation-like domination. They find that simplified analysis based on harmonic behavior of the zero mode, giving rise to a Mathieu equation for the non-zero modes miss important physics. Furthermore, analysis that do not include the full backreaction do not conserve energy, resulting in unbound particle production. Their results do not support the recent claim of symmetry restoration by non-equilibrium fluctuations. Finally estimates of the reheating temperature are given, as well as a discussion of the inconsistency of a kinetic approach to thermalization when a non-perturbatively large number of particles is created.

Yamammoto *et al* [362] (1996) first develop a method to calculate a complete set of mode functions which describe the quantum fluctuations generated in one-bubble open inflation models. They consider two classes of models. One is a single scalar field model proposed by Bucher, Goldhaber and Turok and by them as an example of the open inflation scenario, and the other is a two-field model such as the "supernatural" inflation proposed by Linde and Mezhlumian. In both cases they assume the difference in the vacuum energy density between inside and outside the bubble is negligible. There are two kinds of mode functions. One kind has usual continuous spectrum and the other has discrete spectrum with characteristic wavelengths exceeding the spatial curvature scale. The latter can be further divided into two classes in terms of its origin. One is called the de Sitter super-curvature mode, which arises due to the global spacetime structure of de Sitter space, and the other is due to fluctuations of the bubble wall. They calculate the spectrum of quantum fluctuations in these models and evaluate the resulting large angular scale CMB anisotropies. They find there are ranges of model parameters that are consistent with observed CMB anisotropies.

Dine and Riotto [101] (1997) note that inflation, as currently understood, requires the presence of fields with very flat potentials. Supersymmetric models in which supersymmetry breaking is communicated by supergravity naturally yield such fields, but the scales are typically not suitable for obtaining both sufficient inflation and a suitable fluctuation spectrum. In the context of recent ideas about gauge mediation, there are new candidates for the inflaton. They present a simple model for slow-rollover inflation where the vacuum energy driving inflation is related to the same F-term responsible for the spectrum of supersymmetric particles in gauge mediated supersymmetry breaking models. The inflaton is identified with field responsible for the generation of the μ-term. This opens the possibility of getting some knowledge about the low-energy supersymmetric theory from measurements of the cosmic microwave background radiation. They say that gravitinos do not pose a cosmological problem, while the moduli problem is ameliorated.

Guzmán (1997) [159] finds that for the Bianchi types I-II-III-V in the Brans-Dicke theory, the scalar field of the theory ϕ has the same form in the isotropic case. He shows that isotropization of the Universe occurs in a very short time when the Universe is dominated by vacuum energy, proving that an isotropic Robertson-Walker model is a good approximation to use in the extended inflation scenario. Petry (1997) [274] finds at the beginning of the Universe radiation, matter and vacuum energy given by the cosmological constant are zero and then emerge from gravitational energy. In the course of time the energy of radiation and matter decreases whereas the vacuum energy increases for ever.

Berera *et al* [40] (1998) present a quantum field theory warm inflation model is presented that solves the horizon/flatness problems. They say that the model obtains, from the elementary dynamics of particle physics, cosmological scale factor trajectories that begin in a radiation dominated regime, enter an inflationary regime and then smoothly exit back into a radiation dominated regime, with nonnegligible radiation throughout the evolution.

Malik and Wands [237] (1998) investigate the dynamics of the recently proposed model of assisted inflation. In this model an arbitrary number of scalar fields with exponential potentials evolve towards an inflationary scaling solution, even if each of the individual potentials is too steep to support inflation on its own. By choosing an appropriate rotation in field space they can write down explicitly the potential for the weighted mean field along the scaling solution and for fields orthogonal to it. This demonstrates that the potential has a global minimum along the scaling solution. They show that the potential close to this attractor in the rotated field space is analogous to a hybrid inflation model, but with the vacuum energy having an exponential dependence upon a dilaton field. They present analytic solutions describing homogeneous and inhomogeneous perturbations about the attractor solution without resorting to slow-roll approximations. They discuss the curvature and isocurvature perturbation spectra produced from vacuum fluctuations during assisted inflation.

Starkman *et al* [327] (1999) note that current observations of Type Ia supernovae provide evidence for cosmic acceleration out to a redshift of $z \sim 1$, leading to the possibility that the universe is entering an inflationary epoch. However, inflation can take place only if vacuum-energy (or other sufficiently slowly redshifting source of energy density) dominates the energy density of a region of physical radius 1/H. They argue that for the best-fit values of Ω_Λ and Ω_m inferred from the supernovae data, one must confirm cosmic acceleration out to at least $z \simeq 1.8$ to infer that the universe is inflating.

For Guendelman's [155] (2000) recent discussion of inflation, see §2.16 above,and [382, 383, 384].

4.5 Vacuum Energy as Critical Density.

Vacuum energy can also be considered to take what Sciama, see §4.1¶2 would consider to be a "non-symmetric" form as just being a density. More specifically it can be considered to be background critical density. See also Sokolov [322] (1994) and §3.6. Some recent papers on this include those below.

Kosowsky and Turner [203] (1992) introduce an approximation to calculate the gravitational radiation produced by the collision of true-vacuum bubbles that is simple enough to allow the simulation of a phase transition by the collision of hundreds of bubbles. This "envelope approximation" neglects the complicated "overlap" regions of colliding bubbles and follows only the evolution of the bubble walls. The approximation accurately reproduces previous results for the gravitational radiation from the collision of two scalar-field vacuum bubbles. Using a bubble nucleation rate given by $\Gamma = \Gamma_0 e^{\beta t}$, they simulate a phase transition by colliding 20 to 200 bubbles; the fraction of vacuum energy released into gravity waves is $E_{\rm GW}/E_{\rm vac} = 0.06(H/\beta)^2$ and the peak of the spectrum occurs at $\omega_{\max} = 1.6\beta$ ($H^2 = 8\pi G\rho/3$ is the Hubble constant associated with the false-vacuum phase). The spectrum is very similar to that in the two-bubble case, except that the efficiency of gravity-wave generation is about five times higher, presumably due to the fact that a given bubble collides with many others. Finally, they consider two further "statistical" approximations, where the gravitational radiation is computed as an incoherent sum over individual bubbles weighted by the distribution of bubble sizes. These approximations provide reasonable estimates of the gravitational-wave spectrum with far less computation.

John and Babu [185] (1996) discuss a modified version of the Özer-Taha [264] (1986) nonsingular cosmological model. John and Babu assume that the universe's radius is complex if it is regarded as empty, but that it contains matter when the radius is real. Their model predicts the values: $\Omega_M \equiv \rho_M/\rho_C \approx 4/3$, $\Omega_V \equiv \rho_V/\rho_C \approx 2/3$ and $\Omega_- \equiv \rho_-/\rho_c << 1$ in the present non-relativistic era, where ρ_m =matter density, ρ_V =negative energy density, not necessarily the cosmological constant, but perhaps a spacetime dependent cosmological constant in the style of Chen and Wu [75] (1990), and ρ_C =critical density.

Lima [227] (1996) derives a new Planckian distribution for cosmologies with photon creation using thermodynamics and semiclassical considerations. This spectrum is preserved during the evolution of the universe and compatible with the present spectral shape of the cosmic microwave background radiation(CMBR). Accordingly, the widely spread feeling that cosmologies with continuous photon creation are definitely ruled out by the COBE limits on deviation of the CMBR spectrum from blackbody shape should be reconsidered. It is argued that a crucial test for this kind of cosmologies is provided by measurements of the CMBR temperature at high redshifts. For a given redshift z greater than zero, the temperature is smaller than the one predicted by the standard FRW model.

Adams and Laughlin [5] (1997) outline astrophysical issues related to the long term fate of the universe. They consider the evolution of planets, stars, stellar populations, galaxies, and the universe itself over time scales which greatly exceed the current age of the universe.

Beane [34] (1997) notes that general arguments suggest the existence of at least one unobserved scalar particle with Compton wavelength bounded from below by one tenth of a millimeter, if the mechanism responsible for the smallness of the vacuum energy is consistent with local quantum field theory. He shows that this bound is saturated if vacuum energy is a substantial component of the energy density of the universe. Therefore, the success of cosmological models with a significant vacuum energy component suggests the existence of new macroscopic forces with range in the sub-millimeter region. There are virtually no experimental constraints on the existence of quanta with this range of interaction.

Gentry [145] (1997) notes that a nonhomogeneous universe with vacuum energy, but without spacetime expansion, is utilized together with gravitational and Doppler redshifts as the basis for proposing a new interpretation of the Hubble relation and the 2.7K cosmic blackbody radiation.

Arbab [18] (1999) finds the universe is found to have undergone several phases in which the gravitational constant had a different behaviour. During some epoch the energy density of the universe remained constant and the universe remained static. In the radiation dominated epoch the radiation field satisfies the Stefan's formula while the scale factor varies linearly with. The model enhances the formation of the structure in the universe as observed today.

Hogan [176] (1999) presents a brief but broad survey is presented of the flows, forms and large-scale transformations of mass-energy in the universe, spanning a range of about twenty orders of magnitude (m_{Planck}/m_{proton}) in space, time and mass. Forms of energy he considers include electromagnetic radiation, magnetic fields, cosmic rays, gravitational energy and gravitational radiation, baryonic matter, dark matter, vacuum energy, and neutrinos; sources considered include vacuum energy and cosmic expansion, fluctuations and gravitational collapse, AGN and quasars, stars, supernovae and gamma ray bursts.

Kujat and Scherrer [211] (1999) note that a time variation in the Higgs vacuum expectation value alters the electron mass and thereby changes the ionization history of the universe. This change produces a measurable imprint on the pattern of cosmic microwave background (CMB) fluctuations. The nuclear masses and nuclear binding energies, as well as the Fermi coupling constant, are also altered, with negligible impact on the CMB. They calculate the changes in the spectrum of the CMB fluctuations as a function of the change in the electron mass. They find that future CMB experiments could be sensitive to —$\Delta m_e/m_e| \sim |\Delta G_F/G_F| \sim 10^{-2} - 10^{-3}$. However, they also show that a change in the electron mass is nearly, but not exactly, degenerate with a change in the fine-structure constant. If both the electron mass and the fine-structure constant are time-varying, the corresponding CMB limits are much weaker, particularly for $l < 1000$.

Pascual-Sáchez [269] (1999) develops an alternative explanation for the acceleration of the cosmic expansion, which seems to be a result of recent high redshift Supernova data. In the current interpretation, this cosmic acceleration is explained by including a positive cosmological constant term (or vacuum energy), in the standard Friedmann models. Instead, he considers a locally rotationally symmetric (LRS) and spherically symmetric (SS), but inhomogeneous spacetime, with a barotropic perfect fluid equation of state for the cosmic matter. The congruence of matter has acceleration, shear and expansion. Within this framework the kinematical acceleration of the cosmic fluid or, equivalently, the inhomogeneity of matter, is just the responsible of the SNe Ia measured cosmic acceleration. Although in his model the cosmological principle is relaxed, it maintains almost isotropy about our worldline in agreement with CBR observations.

Zimdahl *et al* (2000) explain an accelerated expansion of the present universe, suggested from observations of supernovae of type Ia at high redshift, by introducing an anti-frictional force that is self-consistently exerted on the particles of the cosmic substratum. Cosmic anti-friction, which is intimately related to "particle production", is shown to give rise to an effective negative pressure of the cosmic medium. While other explanations for an accelerated expansion (cosmological constant, quintessence) introduce a component of dark energy besides "standard" cold dark matter (CDM) they resort to a phenomenological one-component model of CDM with internal self-interactions. They demonstrate how the dynamics of the ΛCDM model might be recovered as a special case of cosmic anti-friction. The connection with two-component models is discussed, providing a possible phenomenological solution to the coincidence problem.

4.6 Equating Vacuum Energy with Dark Matter.

Vacuum energy being non-tangible and (excluding the Casimir effect) so far non-measurable is idealy equated with dark matter which is likewise. Another way of thinking about the quantum vacuum is that it might have large scale effects, it could be just a different way of referring to dark matter - albeit with a different physical interpretation. Some recent papers which can be thought of as equating vacuum energy with dark matter include the following.

Spindel and Brout [324] (1993) use as dynamical variable the square of the radius of the Universe, they solve analytically the Einstein equations in the framework of Robertson-Walker models where a cosmological constant describing phenomenologically the vacuum energy decays into radiation. Emphasis is put on the computation of the entropy creation.

Abdel-Rahman [1] (1995) introduces a nonsingular closed universe model with continuous creation of radiation or matter from the vacuum. Although primordial nucleosynthesis in the model follows the standard scenario it does not require the density of the baryonic matter to be below the critical density as in standard cosmology. The model predicts a present a present vacuum energy comparable with matter energy. Its predictions for classical low red-shift cosmological tests agree with the standard flat model results.

Lima and Maia [391] (1995) deduce the spectrum of a pure vacuum state, assuming that it is formed by a kind of radiation satisfying the equation of state $p = -\rho$. Actually, in this paper they deduce the spectrum for a large class of massless particles satisfying the equation of state $p = (\gamma - 1)\rho$. Particular attention is given for the vacuum case ($\gamma = 0$) and its thermodynamic behavior in their section V. From a historical viewpoint, the introduction is also of interest.

The vacuum spectrum deduced here is completely different of the one appearing in the zero-point approach of stochastic electrodynamics. Some consequences for cosmology are discussed in in their section VII.

Grøn and Soleng [153] (1996) note that on the scales of galaxies and beyond there is evidence for unseen dark matter. In this paper they find the experimental limits to the density of dark matter bound in the solar system by studying its effect upon planetary motion. Roberts [286] (1987) has studied how the cosmological constant might alter the orbit of Pluto. Van Flandern [398] (1999) has also studied possible connections between dark matter and solar system dynamics.

Pfenning and Ford [275] (1996) develop quantum inequality restrictions on the stress-energy tensor for negative energy for three and four-dimensional static spacetimes. They derive a general inequality in terms of a sum of mode functions which constrains the magnitude and duration of negative energy seen by an observer at rest in a static spacetime. This inequality is evaluated explicitly for a minimally coupled scalar field in three and four-dimensional static Robertson-Walker universes. In the limit of vanishing curvature, the flat spacetime inequalities are recovered. More generally, these inequalities contain the effects of spacetime curvature. In the limit of short sampling times, they take the flat space form plus subdominant curvature-dependent corrections.

Antonsen and Bormann [17] (1998) note that for a Friedman-Robertson-Walker spacetime in which the only contribution to the stress-energy tensor comes from the renormalised zero-point energy (i.e. the Casimir energy) of the fundamental fields the evolution of the universe (the scale factor) depends upon whether the universe is open, flat or closed and upon which fundamental fields inhabit the space-time. They calculate this "Casimir effect" using the heat kernel method, and the calculation is thus non-perturbative. They treat fields of spin $0, 1/2, 1$ coupled to the gravitational background only. The heat kernels and/or ζ-functions for the various spins are related to that of a non-minimally coupled one. A WKB approximation is used in obtaining the radial part of that heat kernel. The simulations of the resulting equations of motion seem to exclude the possibility of a closed universe, $K = +1$, as these turn out to have an overwhelming tendency towards a fast collapse - the details such as the rate of this collapse depends on the structure of the underlying quantum degrees of freedom: a non-minimal coupling to curvature accelerates the process. Only $K = -1$ and K=0 will in general lead to macroscopic universes, and of these $K = -1$ seems to be more favourable. The possibility of the scale factor being a concave rather than a convex function potentially indicates that the problem of the large Hubble constant is non-existent as the age of the universe need not be less than or equal to the Hubble time. Note should be given to the fact, however, that we are not able to pursue the numerical study to really large times neither do simulations for a full standard model.

Özer [263] (1999) shows that in the cosmological models based on a vacuum energy decaying as a^{-2}, where a is the scale factor of the universe, the fate of the universe in regard to whether it will collapse in future or expand forever is determined not by the curvature constant k but by an effective curvature constant k_{eff}. He argues that a closed universe with k=1 may expand forever, in other words simulate the expansion dynamics of a flat or an open universe because of the possibility that $k_{eff} = 0$ or -1, respectively. Two such models, in one of which the vacuum does not interact with matter and in another of which it does, are studied. He shows that the vacuum equation of state $p_{vac} = -\rho_{vac}$ may be realized in a decaying vacuum cosmology provided the vacuum interacts with matter. The optical depths for gravitational lensing as a function of the matter density and other parameters in the models are calculated at a source redshift of 2. The age of the universe is discussed and shown to be compatible with the new Hipparcos lower limit of 11Gyr. He suggests the possibility that a time-varying vacuum energy might serve as dark matter.

Turner [341] (1999) notes that more than sixty years ago Zwicky made the case that the great clusters of galaxies are held together by the gravitational force of unseen (dark) matter. Today, he claims that the case is stronger and more precise; I disagree and prefer dynamical explanations. Turner says that dark, nonbaryonic matter accounts for 30% +/- 7% of the critical mass density, with baryons (most of which are dark) contributing only 4.5% +/- 0.5% of the critical density. The large-scale structure that exists in the Universe indicates that the bulk of the nonbaryonic dark matter must be cold (slowly moving particles). The Super Kamiokande detection of neutrino oscillations shows that particle dark matter exists, crossing an important threshold. Over the past few years a case has developed for a dark-energy problem. This dark component contributes about 80% +/- 20% of the critical density and is characterized by very negative pressure ($p_X < -0.6 rho_X$). Consistent with this picture of dark energy and dark matter are measurements of CMB anisotropy that indicate that total contribution of matter and energy is within 10% of the critical density. Fundamental physics beyond the standard model is implicated in both the dark matter and dark energy puzzles: new fundamental particles (e.g., axion or neutralino) and new forms of relativistic energy (e.g., vacuum energy or a light scalar field). Note that here Turner is not necessarily equating vacuum energy with the cosmological constant. A flood of observations will shed light on the dark side of the Universe over the next two decades; as it does it will advance our understanding of the Universe and the laws of physics that govern it.

4.7 Sonoluminescence.

Sonoluminescence was first found to occur when degassed water is irradiated by ultra-sound, Frenzel and Schultes [132] (1934). A stable sonoluminescence can be contrived with a bubble that is trapped at the pressure anti-node of a standing sound wave in a spherical or cylindrical container and that collapses and reexpands with the periodicity of the sound. Barber and Putterman [25] (1991) note that sonoluminescence is a non-equilibrium phenomenon in which the energy in a sound wave becomes highly concentrated so as to generate flashes of light in a liquid. They show that these flashes, which comprise of over 10^5 photons, are too fast to be resolved by the fastest photomultiplier tubes available. Furthermore, when sonoluminescence is driven by a resonant sound field, the bursts can occur in a continuous repeating, regular fashion. These precise 'clock-like' emissions can continue for hours at drivefrequencies ranging from audible to ultrasonic. These bursts represent an amplification of energy by eleven orders of magnitude. A light pulse is emittied during every cycle of the sound wave see Gaitan *et al* [140](1992). Schwinger [314] (1994) suggested that the mechanism responsible for radiation in sonoluminescence is a dynamical version of the Casimir effect, compare §3.5. Boundaries are an essential ingredient of the Casimir effect and in the present case Schwinger takes them to be given by the boundary between a dielectric medium (the water) and the vacuum (the gas inside the bubble). Perhaps the Universe, or more accurately a large segment of ther Universe, can be modeled by the vibrating bubble; in that case the study of sonoluminescence is an analog of the study of QFT in cosmological backgrounds that can be subject to empirical testing. A vibrating bubble analog of the Universe might have application in the study of discrete red shift, compare Roberts [300] (2000).

Eberlein [110] (1995) explains sonoluminescence in terms of quantum radiation by moving interfaces between media of different polarizability. It can be considered as a dynamic Casimir effect, in the sense that it is a consequence of the imbalance of the zero-point fluctuations of the electromagnetic field during the non-inertial motion of a boundary. The transition amplitude from the vacuum into a two-photon state is calculated in a Hamiltonian formalism and turns out to be governed by the transition matrix-element of the radiation pressure. She gives expressions for the spectral density and the total radiated energy.

Chodos [80] (1996) notes that the phenomenon of sonoluminescence (SL), originally observed some sixty years ago, has recently become the focus of renewed interest, particularly with the discovery that one can trap a single bubble and induce it to exhibit SL stably over a large number of acoustical cycles. In this work he adopts a version of the provocative suggestion put forward by Schwinger: the mechanism responsible for the radiation in SL is a dynamic version of the Casimir effect. It has been known since Casimir's original work in [69] (1948) that the zero-point energy of quantum fields can be modified by the presence of boundaries, and that these modifications generate observable effects. For example, in Casimir's original work, the quantum fluctuations of the electromagnetic field in the presence of a pair of uncharged, parallel, perfectly conducting plates were shown to give rise to an attractive force between the plates.

Liberati *et al* [224] (1999) investigate several variations of Schwinger's proposed mechanism for sonoluminescence. They demonstrated that any realistic version of Schwinger's mechanism must depend on extremely rapid (femtosecond) changes in refractive index, and discussed ways in which this might be physically plausible. To keep that discussion tractable, the technical computations in their earlier paper were limited to the case of a homogeneous dielectric medium. In their later paper they investigate the additional complications introduced by finite-volume effects. The basic physical scenario remains the same, but now there are finite spherical bubbles, and so must decompose the electromagnetic field into spherical harmonics and Bessel functions. They demonstrate how to set up the formalism for calculating Bogolubov coefficients in the sudden approximation, and show that qualitatively the results previously obtained using the homogeneous-dielectric (infinite volume) approximation are retained.

See also Milton [246] (2000) §3.5.

4.8 Quantum Cosmology.

Quantum cosmology requires notions of a vacuum from all *three* of the previous sections. From the *first* because it is a quantum field theory; from the *second*, specifically §3.9, because it is on curved spacetime; and from the *third* because it is concerned with large scales. Some recent papers on quantum cosmology that mention the vacuum include those mentioned below.

Brevik *et al* [54] (2000) apply the background field method and the effective action formalism to describe the four-dimensional dynamical Casimir effect. Their picture corresponds to the consideration of quantum cosmology for an expanding FRW universe (the boundary conditions act as a moving mirror) filled by a quantum massless GUT which is conformally invariant. They consider cases in which the static Casimir energy is repulsive and attractive. Inserting the simplest possible inertial term, they find, in the adiabatic (and semiclassical) approximation, the dynamical evolution of the scale factor and the dynamical Casimir stress analytically and numerically (for SU(2) super Yang-Mills theory). They also explore alternative kinetic energy terms.

Fink and Leschke [124] (2000) note that in order to relate the probabilistic predictions of quantum theory uniquely to measurement results, one has to conceive of an ensemble of identically prepared copies of the quantum system under study. Since the universe is the total domain of physical experience, it cannot be copied, not even in a thought experiment. Therefore, a quantum state of the whole universe can never be made accessible to empirical test. Hence the existence of such a state is only a metaphysical idea. Despite prominent claims to the contrary, recent developments in the quantum-interpretation debate do not invalidate this conclusion.

Padmanabhan and Choudhury [265] (2000) note that starting from an unknown quantum gravitational model, one can invoke a sequence of approximations to progressively arrive at quantum field theory (QFT) in curved spacetime, QFT in flat spacetime, nonrelativistic quantum mechanics and newtonian mechanics. The more exact theory can put restrictions on the range of possibilities allowed for the approximate theory which are not derivable from the latter - an example being the symmetry restrictions on the wave function for a pair of electrons. They argue that the choice of vacuum state at low energies could be such a 'relic' arising from combining the principles of quantum theory and general relativity, and demonstrate this result in a simple toy model. Their analysis suggests that the wave function of the universe, when it describes the large volume limit of the universe, dynamically selects a vacuum state for matter fields - which in turn defines the concept of particle in the low energy limit. The result also has the potential for providing a concrete quantum mechanical version of Mach's principle.

5 Principle of Equivalence

Misner Thorne and Wheeler [247] page 386 (1972) formulate the *principle of equivalence* as follows

> In any and every local Lorentz frame, anywhere and anytime in the Universe, all (nongravitational) laws of physics must take on their familiar special-relativistic forms.

One can take this view as implying that the expectation value of non-gravitational fields is the same in every frame. This can be viewed as a symmetry requirement on the vacuum which can require that it produces a cosmological constant, compare Sciama's [310] argument discussed here §4.1¶2.

Barut and Haugen [29] (1972) produce a theory where mass is also conformally invariant, which might allow the requirement of a local Loretz frame to be relaxed.

Alvarez and Mann [13] (1996) consider possible tests of the Einstein Equivalence Principle for quantum-mechanical vacuum energies by evaluating the Lamb shift transition in a class of non-metric theories of gravity described by the τ formalism. They compute to lowest order the associated red shift and time dilation parameters, and discuss how (high-precision) measurements of these quantities could provide new information on the validity of the equivalence principle.

Bertolami and Carvalho [370] (2000) show in the context of a Lorentz-violating extension of the standard model that estimates of Lorentz symmetry violation extracted from ultra-high energy cosmic rays beyond the Greisen-Kuzmin-Zatsepin (GZK) cutoff allow for setting bounds on parameters of that extension. Furthermore, they argue that a correlated measurement of the difference in the arrival time of gamma-ray photons and neutrinos emitted from active galactic nuclei or gamma-ray bursts may provide a signature of possible violation of Lorentz symmetry. They have found that this time delay is energy independent, however it has a dependence on the chirality of the particles involved. They also briefly discuss the known settings where the mechanism for spontaneous violation of Lorentz symmetry in the context of string/M-theory might take place.

Leung [222] (2000) reviews the status of testing the principle of equivalence and Lorentz invariance from atmospheric and solar neutrino experiments.

Lyre [235] (2000) discusses generalizing the principle of equivalence, as does Roberts [290] (1989), who argues that the Higgs mechanism, see §2.10, is not compatible with a general principle of equivalence.

Nikolic [257] (2000) discusses the nature of acceleration, which by the equivalence principle is related to the gravitational field. Whether there is a maximal acceleration is reviewed in Papini [267] (1995).

6 Energy In General Relativity.

6.1 Does Vacuum Energy have Gravitational Effects?

If vacuum energy is non-zero then it will have a gravitational effect via field equations equating geometry to matter. The vacuum energy of non-gravitational fields will contribute to matter; whether there is a contribution to the geometry side via "vacuum energy from geometry" depends upon how this is viewed. From the point of view of supersymmetric theories there is and the terms cancel out term by term, see §2.6. At this point it is necessary to address what, if anything, is the meaning of the semi-classical equations

$$G_{ab} =< T_{ab} > \tag{6.1}$$

equation the Einstein tensor to the expected value $<>$ of the stress T_{ab}, c.f. §3.9. One could view the lowest value of this expectation as being the vacuum value, then a non-zero vacuum expectation implies that the Ricci-flat equations $R_{ab} = 0$ never occur in nature, compare Roberts [283] (1985). There is the question of what $< G_{ab} >$ could mean, and this is part of the subject of quantum gravity.

Sciama's [310] view on the relationship between zero-point energy and gravitation is now presented. If an energy $\frac{1}{2}h\nu$ is ascribed to each mode of the vacuum radiation field, then the total energy of the vacuum is infinite. It would clearly be inconsistent with the original assumption of a background Minkowski spacetime to suppose that this energy produces gravitation in a manner controlled by Einstein's field equations of general relativity. It is also clear that the spacetime of the real world approximates closely to the Minkowski state, at least on macroscopic scales. It thus appears that must be regularized the zero-point energy of the vacuum by subtracting it out according to some systematic prescription, and this is one way of looking at QFT's on curved spaces, see §3.9, and for example Baym [32] (1994) discusses how to locate renormalized energy in Robertson-Walker spacetimes. At the same time, it would expected that zero-point energy *differences* gravitate. For example, the (negative) Casimir energy between two plane-parallel perfect conductors would be expected to gravitate; otherwise, the relativistic relation between measured energy and gravitation would be lost. Similarly, the regularized vacuum energy in a curved spacetime would be expected to gravitate, where the regularization is achieved by subtracting out the Minkowski contribution in a systematic way. This procedure is needed in order to obtain a pragmatically workable theory. The difficulty with it is that existing theory does not tell which is the fiducial state whose energy is to be set to zero. Sciama [310] claims that: "It is no doubt an intelligent guess that one should take Minkowski spacetime as this fiducial state, but the awkward point is that this (or any other) choice is not *prescribed* by existing theory. Clearly, something essential is missing."

I think that Sciama is wrong, at least for QFT's on curved spaces, the correct procedure is to normalize a stress relative to its Minkowski value. Another way of looking at this problem is in terms of the cosmological constant, see §4.

Datta [92] (1995) discusses an interesting development in semiclassical gravity. Using an improved Born-Oppenheimer approximation, the semi-classical reduction of the Wheeler-deWitt equation turns out to give important insights into the nature and the level of validity of the semi-classical Einstein equations (SCEE). He shows back reactions from the quantized matter fields in SCEE to be completely determined by adiabatically induced U(N) gauge potentials. The finite energy from the vacuum polarization, in particular, is found to be intimately related to the 'magnetic' type geometric gauge potential. As a result the vacuum energy in a universe from a 'source-free' flat simply connected superspace is gauge equivalent to zero, leading to some dramatic consequences.

6.2 Various Approaches to Gravitational Energy.

Gravitational energy is unusual in that it is usually negative. This means that gravitational energy can cancel out stresses T_{ab} which obey good energy conditions. An example of this is the imploding scalar-Einstein solution, Roberts [289, 294] (1989,1996) where the positive energy of the scalar field and the negative energy of the gravitational field cancel out. Such a situation might happen for vacuum energy, the energy of the gravitational field might cancel it out. To gain insight into this it is necessary to study the energy of the gravitational field.

The energy of the gravitational field is unlike that of other fields as it is not usually represented by a tensor. If a non-tensorial expressions is chosen there exists a coordinate system in which the expression can be transferred to zero. A spacetime might have a preferred vector field, this extra information can allow non-tensorial expressions to be given an invariant meaning. For example for asymptotically flat spacetimes there is a preferred vector field normal to the 3-sphere at infinity which allows construction of an expression for total energy measured at infinity. In spacetimes which are not asymptotically flat there are several ways of approaching gravitational energy. Here the two-point approach is investigated by looking at its form in deSitter spacetime. An observer might not be able to measure energy at a single point but there is the possibility of measuring an energy difference between two points. Two-point energy expressions have been investigated by Synge and Lanthrop, see the next paragraph.

Gravitational energy is definable for asymptotically flat spacetime. For spherically symmetric spacetimes there is an expression de Oliveria and Cheb-Terrab [97] (1996) which reduces to the correct expression for an asymptotically flat spacetime and which agrees with quasi-local expressions. In general there are many, not necessarily compatible, ways of defining gravitational energy. Some approaches to gravitational energy in arbitrary spacetime are: *one* to construct pseudo-tensors Goldberg [150] (1958), *two* to construct tensors from the Riemann tensor such as the tensors of Bel [35] (1962) (see also Alder *et al* [8](1977) Appendix E) , and produce 'square root' tensors from these of the correct dimensions Bonilla and Senovilla [47] (1997) and Bergqvist [41] (1998) , *three* to construct tensors from the Lanczos tensor Roberts [288] (1988), these have been used to measure the speed of energy transfer in gravitational waves Roberts [293] (1994), *four* to construct quasi-local expressions such as those of Brown and York [58] (1993), Hayward [168] (1995) gives examples of these, *five* general relativity can be re-written as a tele-parallel theory and in this theory there is a tensorial expression for gravitational energy Maluf [238] (1995), *six* to construct Synge [332, 333] (1960/2)-Lathrop [218] (1975) two-point expressions. Other two-point approaches are those of Droz-Vincent [105] (1996) who compares the energy of an exploding spacetime to that at the origin of the coordinates, and of Isaacson [180] (1968) who compares the energy of gravitational waves at two separate times. Babak and Grishchuk [22] (2000), also study the energy-momentum tensor for the gravitational field. One can also ask if there is a maximum gravitational energy, which is related, by the equivalence principle, see §5, to the question of whether there is a maximum acceleration, see the review of Papini [267] (1995). For the relation of acceleration to quantum localization, see Jaekel and Reynaud [183] (1999).

Brown *et al* [57] (1998) consider the definition E of quasilocal energy stemming from the Hamilton-Jacobi method as applied to the canonical form of the gravitational action. They examine E in the standard "small-sphere limit," first considered by Horowitz and Schmidt in their examination of Hawking's quasilocal mass. By the term "small sphere" they mean a cut $S(r)$, level in an affine radius r, of the lightcone belonging to a generic spacetime point. As a power series in r, they compute the energy E of the gravitational and matter fields on a spacelike hypersurface spanning $S(r)$. Much of Their analysis concerns conceptual and technical issues associated with assigning the zero-point of the energy. For the small-sphere limit, they argue that the correct zero-point is obtained via a "lightcone reference," which stems from a certain isometric embedding of $S(r)$ into a genuine lightcone of Minkowski spacetime. Choosing this zero-point, we find agreement with Hawking's quasilocal mass expression, up to and including the first non-trivial order in the affine radius. The vacuum limit relates the quasilocal energy directly to the Bel-Robinson tensor. They present a detailed examination of the variational principle for metric general relativity as applied to a "quasilocal" spacetime region M (that is, a region that is both spatially and temporally bounded). Our analysis relies on the Hamiltonian formulation of general relativity, and thereby assumes a foliation of M into spacelike hypersurfaces Σ. They allow for near complete generality in the choice of foliation. Using a field-theoretic generalization of Hamilton-Jacobi theory, they define the quasilocal stress-energy-momentum of the gravitational field by varying the action with respect to the metric on the boundary ∂M. The gravitational stress-energy-momentum is defined for a two-surface B spanned by a spacelike hypersurface in spacetime. They examine the behavior of the gravitational stress-energy-momentum under boosts of the spanning hypersurface. The boost relations are derived from the geometrical and invariance properties of the gravitational action and Hamiltonian.

Johri *et al* (1995) [186] examines the role of gravity in the evolution of the universe is examined. They claims that in co-moving coordinates, calculation of the Landau-Lifshitz pseudotensor for FRW models reveals that:

i) the total energy of a spatially closed universe irrespective of the equation of state of the cosmic fluid is zero at all times.
ii)the total energy enclosed within any finite volume of the spatially flat universe is zero at all times,
iii)during inflation the vacuum energy driving the accelerated expansion and ultimately responsible for the creation of matter (radiation) in the universe, is drawn from the energy of the gravitational field. In a similar fashion, certain cosmological models which abandon adiabaticity by allowing for particle creation, use gravitational energy directly as an energy source.

Tung and Nester [397] (2000) note that in the tetrad representation of general relativity, the energy-momentum expression, found by Möller in 1961, is a tensor with respect to coordinate transformations but is not a tensor with respect to local Lorentz frame rotations. This local Lorentz freedom is shown to be the same as the six parameter normalized spinor degrees of freedom in the quadratic spinor representation of general relativity. From the viewpoint of a gravitational field theory in flat spacetime, these extra spinor degrees of freedom allow them to obtain a local energy-momentum density which is a true tensor over both coordinate and local Lorentz frame rotations.

7 Inertia.

7.1 Inertia in General.

In non-quantum physics inertia would normally be approached through the principle of equivalence, see §5. What an inertial vacuum is is discussed in Padmanabhan and Choudhury [265] (2000). Ciufolini and Wheeler [81] (1995) also discuss gravitation and inertia. Meissner and Veneziano [244] (1991) discuss what can be thought of as trying to find which frames are inertial.

Bozhkov and Rodrigues (1995) [53] consider the Denisov-Solov'ov (1983) [95] example which some claims shows that the inertial mass is not well defined in general relativity. They show that the mathematical reason why this is true is a wrong application of Stokes theorem. Then they discuss the role of the order of asymptotically flatness in the definition of mass. In conclusion they present some comment on the conservation laws in general relativity.

Farup and Grøn [120] (1996) investigate if there is any inertial dragging effect associated with vacuum energy. Spacetime inside and outside a rotating thin shell, as well as the mechanical properties of the shell, are analyzed by means of Israel's general relativistic theory of surface layers. Their investigations generalizes that of Brill and Cohen [56] (1966) who found vacuum solutions of Einstein's field equations (with vanishing cosmological constant), inside and outside a rotating shell. They include a non-vanishing vacuum energy inside the shell. They find that the inertial dragging angular velocity increases with increasing density of the vacuum. da Paola and Svaiter [272] (2000) discuss how rotating vacuums have inertia and their relation to Mach's principle.

Directional preferences and zero-point energy are discussed in Winterberg

[359] (1998), a directional preference suggests a non-inertial frame.

Herrera and Martínez [172] (1998) establish for slowly rotating fluids, the existence of a critical point similar to the one found for nonrotating systems. As the fluid approaches the critical point, the effective inertial mass of any fluid element decreases, vanishing at that point and changing sign beyond it. This result implies that the first order perturbative method is not always reliable to study dissipative processes occurring before relaxation. They comment upon physical consequences that might follow from this effect. They use a nonstatic axisymmetric line element.

Milgrom [392] (1998) has suggested in several papers that dynamics is modified to include an acceleration constant so as to explain dynamics on galactic scales and larger. In this paper he suggests that the vacuum gives rise to the acceleration constant and is also responsible for inertia. He says that either cosmology enters local dynamics by affecting the vacuum, and inertia in turn, through the constant of acceleration a_0; or the same vacuum effect enters both the modified dynamics (MOND) through a_0 and cosmology (e.g. through a cosmological constant). He goes on to say for the vacuum to serve as substratum for inertia a body must be able to read in it its non-inertial motion; this indeed it can, by detecting Unruh-type radiation. A manifestation of the vacuum is also seen, even by inertial observers, in a non-trivial universe (marked, e.g., by curvature or expansion). A non-inertial observer in a nontrivial universe will see the combined effect. An observer on a constant-acceleration (a) trajectory in a de Sitter universe with cosmological constant Λ sees Unruh radiation of temperature $T \propto [a^2 + a_0^2]^{1/2}$, with $a_0 = (\Lambda/3)^{1/2}$. The temperature excess over what an inertial observer sees, $T(a) - T(0)$, turns out to depend on a in the same way that MOND inertia does. An actual inertia-from-vacuum mechanism is still a far cry off.

Capozziello and Lambiase [65] (1999) calculate the inertial effects on neutrino oscillations induced by the acceleration and angular velocity of a reference frame. Such effects have been analyzed in the framework of the solar and atmospheric neutrino problem.

De Paola and Svaiter [272] (2000) consider a quantum analog of Newton's bucket experiment in a flat spacetime: they take an Unruh-DeWitt detector in interaction with a real massless scalar field. They calculate the detector's excitation rate when it is uniformly rotating around some fixed point and the field is prepared in the Minkowski vacuum and also when the detector is inertial and the field is in the Trocheries-Takeno vacuum state. They compare these results and discuss the relations with a quantum analog of Mach's principle.

Haisch *et al* [160] (1994), Haisch and Rueda [161] (1999) and Haisch *et al* [162] (2000) note that even when the Higgs particle is finally detected, it will continue to be a legitimate question to ask whether the inertia of matter as a reaction force opposing acceleration is an intrinsic or extrinsic property of matter. General relativity specifies which geodesic path a free particle will follow, but geometrodynamics has no mechanism for generating a reaction force for deviation from geodesic motion. They discuss a different approach involving the electromagnetic zero-point field (ZPF) of the quantum vacuum. It has been found that certain asymmetries arise in the ZPF as perceived from an accelerating reference frame. In such a frame the Poynting vector and momentum flux of the ZPF become non-zero. Scattering of this quantum radiation by the quarks and electrons in matter can result in an acceleration-dependent reaction force. Both the ordinary and the relativistic forms of Newton's second law, the equation of motion, can be derived from the electrodynamics of such ZPF-particle interactions. They give conjectural arguments are given why this interaction should take place in a resonance at the Compton frequency, and how this could simultaneously provide a physical basis for the de Broglie wavelength of a moving particle. This affords a suggestive perspective on a deep connection between electrodynamics, the origin of inertia and the quantum wave nature of matter.

Modanese [248] (2000) recalls different ways to define inertial mass of elementary particles in modern physics, he then studies the relationship between the mass of charged particles and zero-point electromagnetic fields. To this end he first introduce a simple model comprising a scalar field immersed in stochastic or thermal electromagnetic fields. Then he sketches the main steps of Feynman mass renormalization procedure. His approach is essentially pedagogical and in line with the standard formalism of quantum field theory, but he also tries to keep an open mind concerning the physical interpretation. He checks, for instance, if it is possible to start from a zero bare mass in the renormalization process and express the finite physical mass in terms of a cut-off. Finally he briefly recall the Casimir-induced mass modification of conducting or dielectric bodies.

Nikolic [257] (2000) studies the role of acceleration in the twin paradox. From the coordinate transformation that relates an accelerated and an inertial observer he finds that, from the point of view of the accelerated observer, the rate of the differential lapses of time depends not only on the relative velocity, but also on the product of the acceleration and the distance between the observers. However, this result does not have a direct operational interpretation because an observer at a certain position can measure only physical quantities that are defined at the same position. For local measurements, the asymmetry between the two observers can be attributed to the fact that noninertial coordinate systems, contrary to inertial coordinate systems, can be correctly interpreted only locally.

Rosu [395] (2000) introduces in a nonrigorous manner, the write-up of the talk extends and updates several sections of the review `gr-qc/9406012`, last updated in 1997. The recently introduced glass analogy for black holes is presented, but in order to have a more detailed picture there is a collection from the literature some useful material related to the violations of the fluctuation-dissipation theorem in glass physics and stationary driven systems. Nonequilibrium effective temperatures from glass irreversible thermodynamics are considered as useful and quite general concepts for noninertial quantum fluctuations, though the analogy is not fully disentangled. Next, the stationary and nonstationary scalar vacuum noises is discussed in some detail and the radiometric nature of the Frenet invariants of the stationary worldlines is emphaised, rather than sticking to the thermal interpretation of the vacuum excitations as apparent for the uniformly accelerated quantum detector. The Hacyan-Sarmiento approach for calculating electromagnetic vacuum physical quantities is discussed next. The application to the circular case led Mane to propose the identification of the Hacyan-Sarmiento zero-point radiation with the ordinary synchrotron radiation, but the issue remains still open. The spin flip synchrotron radiation in the context of Bell and Leinaas proposal is breifly discussed. Jackson's approach showing why this proposal is implausible is included. Finally, there is a short random walk in the literature on the fluctuation-dissipation relationship.

Salehi *et al* (2000) study a model for analyzing the effect of a principal violation of the Lorentz-invariance on the structure of vacuum. The model is based on the divergence theory developed by Salehi (1997). They show that the divergence theory can be used to model an ensemble of particles. The ensemble is characterized by the condition that its members are basically at rest in the rest frame of a preferred inertial observer in vacuum. In this way we find a direct dynamical interplay between a particle and its associated ensemble. They show that this effect can be understood in terms of the interaction of a particle with a relativistic pilot wave through an associated quantum potential.

7.2 Superfluid Analogy.

As with quantum field theory, inertia also has superfluid analogies.

Duan [106] (1993) discusses the inertial mass of moving singularities. Duan and Šimánek [107] (1994)extended to finite temperature the theory of the inertial mass of a fluxon in the type II superconductors, due to the coupling between the fluxon and the lattice deformation. They propose an ansatz for the quasi-particle fraction valid at all temperatures below T_c and solve the associated strain field. For high T_c supercondutors the mass is 10^5 electron mass/cm at low temperature (or at least the same order of magnitude as the electromagnetic inertial mass) and vanishes at T_c, this resolving an outstanding problem of the previous theory.

Mel'nikov [245] (1996) considers the dynamics of titled vortex lines in Josephson coupled layered superconductors in considered within the time dependent Ginzburg-Landau theory. The frequency and angular dependences of the complex valued vortex mobility μ are studied. The components of the viscosity and inertial mass tensors are found to increase essentially for magnetic field orientations close to the layers. For superconducting/normal metal multilayers the frequency (ω) range is shown to exist where the μ^{-1}) value depends logarithmically on ω.

Gaitonde and Ramakrishnan [141] (1997) calculate the inertial mass of a moving vortex in cuprate superconductors.

8 Relativity of Motion in Vacuum.

Nogueira and Maia [258] (1995) show that for a self-interacting mass the scalar field in the geometry of Casimir plates and in $N = m + 1$ spacetime dimensions the renormalized ZPA vanishes. So, it is undetectable via Casimir forces.

Guendelman and Rabinowitz [381] (1996) say...???

Nogueira and Maia [259] (1996) investigate a possible difference between the effective potential and zero-point energy. They define the zero-point ambiguity (ZPA) as the difference between these two definitions of vacuum energy. Using the ζ-function technique, in order to obtain renormalized quantities, they show that ZPA vanishes, implying that both of the above definitions of vacuum energy coincide for a large class of geometries and a very general potential. In addition, they show explicitly that an extra term, obtained by E. Myers some years ago for the ZPA, disappears when a scale parameter μ is consistently introduced in all ζ-functions in order to keep them dimensionless.

Kardar and Golestanian [191] (1997) note that the static Casimir effect describes an attractive force between two conducting plates, due to quantum fluctuations of the electromagnetic (EM) field in the intervening space. *Thermal fluctuations* of correlated fluids (such as critical mixtures, super-fluids, liquid crystals, or electrolytes) are also modified by the boundaries, resulting in finite-size corrections at criticality, and additional forces that effect wetting and layering phenomena. Modified fluctuations of the EM field can also account for the 'van der Waals' interaction between conducting spheres, and have analogs in the fluctuation–induced interactions between inclusions on a membrane. They employ a path integral formalism to study these phenomena for boundaries of arbitrary shape. This allows them to examine the many unexpected phenomena of the dynamic Casimir effect due to moving boundaries. With the inclusion of quantum fluctuations, the EM vacuum behaves essentially as a complex fluid, and modifies the motion of objects through it. In particular, from the mechanical response function of the EM vacuum, they extract a plethora of interesting results, the most notable being: (i) The effective mass of a plate depends on its shape, and becomes anisotropic, (ii) There is dissipation and damping of the motion, again dependent upon shape and direction of motion, due to emission of photons, (iii) There is a continuous spectrum of resonant cavity modes that can be excited by the motion of the (neutral) boundaries. Kardar and Golestanian [387] (1999) discuss whether the vacuum has a 'friction'.

Jaekel [184] (1998) reconsider the question of relativity of motion because of the existence of vacuum fluctuations. His article is devoted to this aim with a main line which can be formulated as follows: "The principle of relativity of motion is directly related to symmetries of quantum vacuum". Keeping close to this statement, he discusses the controversial relation between vacuum and motion. He introduces the question of relativity of motion in its historical development before coming to the results obtained more recently.

Jaekel and Reynaud [183] (1998) define quantum observables associated with Einstein localization in spacetime. These observables are built on Poincare and dilatation generators. Their commutators are given by spin observables defined from the same symmetry generators. Their shifts under transformations to uniformly accelerated frames are evaluated through algebraic computations in conformal algebra. Spin number is found to vary under such transformations with a variation involving further observables introduced as irreducible quadrupole momenta. Quadrupole observables may be dealt with as non commutative polarisations which allow one to define step operators increasing or decreasing the spin number by unity.

9 The Vision Thing.

The solution suggested here to the nature of the vacuum is that Casimir energy can produce short range effects because of boundary conditions, but that at long range there is no overall effect of vacuum energy, unless one considers lagrangians of higher order than Einstein's as vacuum induced. That such higher order lagrangians describe nature is likely because they occur as an approximation to most quantum gravity theories, and the Bach lagrangian part of higher order theory might explain various long range effects, see Roberts [292] (1991). Different reductions of quantum gravity theories produce different ratios of the coupling constants of the resulting higher order lagrangian theories. A constant "cosmological constant" does not exist. A non-constant "cosmological constant" which is really a type of perfect fluid might not be zero. No original calculations are presented in support of this position.

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11 Referencing Style.

The almost alphabetical order of the first references is maintained, the reason being so as not to necessitate rewriting my author index:
http://cosmology.mth.uct.ac.za/ roberts/4hcv/4hurl/home.html.
References 187, 272, 310, 345, & 352 are out of order, 16 small corrections are made to first version references. References for the second version go at the end.

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